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What's Going on in Your School?

A Mathematics Inquiry

By RALEIGH SCHORLING

University High School, Ann Arbor, Mich.

THE Commission on Post-War Plans of the National Council of Teachers of Mathematics collected information relating to mathematical education by an inquiry entitled: "What's Going on in Your School?" This inquiry was printed in three parts respectively,—in the February, April and May, 1947, issues of THE MATHEMATICS TEACHER. Although the Commission no longer exists, the persons who responded to the Inquiry are entitled to a report. This article will therefore attempt to interpret the data that were collected. The response to the Inquiry was far greater than could reasonably be expected. The 136 reports received on Part I described the mathematical programs for 133,121 pupils; and the 358 responses received on Part II reported on the mathematical programs of 174,746 pupils. The responses to Part III were fewer, although, as a matter of fact, they are still coming. It is truly amazing that the various journals of our national societies have not used this simple technique for following the trends in their fields.

At this point the reader may need a word of warning. The pupil samples, large as they are, probably are not truly representative of all kinds of schools. In fact we know that the small high schools are not adequately represented in this report. More than $\frac{2}{3}$ of all high schools are small —with fewer than than two hundred students and less than eight teachers. Only 4% of the responses to Part I were from schools with less than one hundred high school students. Then too, not all geographical areas of the country are adequately represented. To be sure, the reports to Part I come from 36 states, the District of Columbia, and Canada, and the responses to Part II came from 28 states, and the District of Columbia. Nevertheless, it soon became obvious that the reports were coming heavily from those areas of our country in which the National Council has been especially active. There is also some duplication in the three samples for the reason that some of the persons reporting sent data for two and in some cases, for three parts of the Inquiry. There is no implication here that the interpretations do not apply to the states from which few returns were received, for in all probability the problems of mathematical education in California and in Texas are about the same as they are in Michigan and in New Jersey.

It will be helpful if the reader has before

him the three parts of the Inquiry published in THE MATHEMATICS TEACHER as we now proceed to interpret the data.

PART I

The first question reads: "Compare the attitude of administrators toward mathematics with that before the war. Is it now (a) more favorable, (b) less favorable, or (c) about the same"? Forty one per cent say "more favorable," 43% say "about the same," only 1½% "less favorable," and 14½% did not reply. Of the 42% who reported "about the same," a considerable number took the trouble to add that the attitude before the war was already very favorable. Especially gratifying is the fact that only 2% say "less favorable." It would seem that at the time of this inquiry, mathematics was still occupying a "place in the sun."

Question two reads: "What grades are enrolled in your building?" The greatest number (29.4%) of the reports came from the traditional type of high school (grades 9 to 12 inclusive). A rather surprising number (26.4%) came from schools that house the junior and senior high schools (grades 7 to 12) in the same building. Fourteen per cent came from junior high schools (grades 7 to 9), and 13.2% from senior high schools (grades 10 to 12). In the remaining 17% there was a truly astonishing variation of administrative organization—in fact, ten other types were involved.

The third item of the Inquiry developed the holding power of mathematics in grades 7 to 12. This same item appears also as item one in Parts II and III. Hence we may interpret the reports for Part I and note the extent to which the generalizations are supported by data from the other two samples. The main findings are: (1) as reported for Part I, 100% of the pupils in grades 7 and 8 are in some type of mathematics course; as reported for Part II, 99% in these two grades are enrolled in mathematics. Obviously this means that mathematics is a required subject in these schools. (2) as reported for Part I, 92% of the students in the ninth grade take some kind of mathematics course; in the sample for Part II, 86% of the ninth grade pupils take mathematics. (3) In grades 10, 11, and 12, the drop in students taking mathematics is fast: for the first sample the figures show 65% of the tenth grade pupils enrolled in some kind of mathematics course, 38% of the 11th grade, and 26% of the 12th grade. In the second sample the figures are: for the 10th grade 59%, for the 11th grade 33%, and for the 12th grade 31%. (4) As regards the general picture for the senior high school, 44% of the pupils involved in Part I and exactly the same per cent (44) of the pupils in Part II elect some kind of mathematics course. Whether this is the way it ought to be, no one can say. In all probability, a test on functional competence in mathematics would show the situation to be very bad. Some pupils in senior high school may not need much mathematics, but what they need may be crucial to them. The policy of excusing from mathematics the students who have perhaps the greatest need for a bit of mathematical training is, in the judgment of the writer, an over-simplification of our problem. One more fact should be noted. If we are thinking of offering new and better mathematics courses for students whose needs are not met in the traditional sequential courses, then the situation in the senior high school offers a splendid opportunity for the reason that 56% of the students are not enrolled in any kind of mathematics course.

In the fourth item there is an effort to find out what kind of mathematics is being offered in the 7th and 8th grades and the degree to which students achieve mastery of the fundamentals. In the first place, over 91% of the students are taking what the teachers think is *general mathematics.* The trend from formal arithmetic in grades 7 and 8 to general mathematics is one of the most significant changes in the teaching of mathematics in the last 30 years. In about 90% of the schools the teachers think they are including informal geom-

etry which ranges all the way from 1% to 40% of the total mathematics offering in these grades. The last part of the question reads as follows: "What fraction of the pupils entering the 7th grade are adequately prepared in their mastery of common and decimal fractions?" The teachers reporting for 7 schools believe that *none* of their pupils are adequately prepared! In three of the schools, the teachers felt that 80% of their pupils were adequately prepared. Note, however, that 80% is the maximum for any school. The general picture of teachers' opinions is that about two out of three pupils are inadequately prepared in common and decimal fractions when they enter the 7th grade.

The responses to question 5 show that almost exactly two out of three schools reporting offer a double track in the 9th grade and these schools enroll about 70% of all the pupils in the two samples. It is significant to note that some teachers who do not now provide the double track, say that they would like to do so. The second part of question number 5 dealt with the types of mathematics courses that pupils in the 9th grade take. The responses show that about 48% take algebra, 34% take general mathematics, 8% take commercial arithmetic, 5% take shop mathematics, and 5% some other course. These other courses include business arithmetic and remedial mathematics. In the judgment of the writer, the number who take algebra (48%) is probably much too high. My reasons are (1) The per cent of failure is probably high. (2) The number of persons taking algebra in the 10th grade probably includes a considerable number that fail in the 9th grade. (3) The recent report of the President's Scientific Research Board[1] does not suggest that it would be necessary for 48% of the 9th grade students to take algebra in order to equip this nation with adequate scientific personnel. A sensible number taking 9th grade algebra in a good school might, I should think, range from 20% to 30%.

In response to item 6, 38% of the schools report that they try to classify pupils according to ability in the 9th grade. Also 81% of the schools that have such groups, provide the double track. Of the schools not offering the double track, only 9% reported that they have ability grouping. The schools providing neither the double track nor ability grouping enroll 18% of the total number of pupils.

In item 7 there is an effort to find out whether schools offer a double track in the 10th grade, and what mathematics courses 10th grade pupils take. About 48% of the schools enrolling 60% of the pupils, say they offer a double track in the 10th grade. What mathematics do 10th grade pupils take? In these schools, 46% take first year algebra, 11% study first year general mathematics, 6% enroll in second year general mathematics, only 22% take demonstrative geometry, 7% are in commercial arithmetic, and 8% in a variety of other courses. The fact that about 1 out of every 9 pupils in the 10th grade is enrolled in first year general mathematics is noteworthy, and the fact that about 1 out of every 16 is taking second year general mathematics is even more significant for the reason that we have few, if any, textbooks specifically designed to provide a second year of general mathematics. The number enrolled in demonstrative geometry (22%) seems low. If about 25% are needed for scientific personnel, then students who are ready for training in formal geometry ought not to delay it. The road to leadership in science and mathematics is so long and so rugged, that no time should be wasted in the senior high school. To be sure, the future scientist may turn out to be the outstanding person in his field, but nevertheless, he can never regain any time that is lost on the road. As has been suggested, about 8% of the students are scattered in a variety of courses that include second year algebra, shop mathematics, business mathematics,

[1] *Manpower for Research*, Volume Four of Science and Public Policy. A Report to the President, by the President's Scientific Research Board. Washington, Government Printing Office, 1947.

bookkeeping, and vocational mathematics. Naturally each of these categories includes only a few schools. The general picture for these schools is that about one half of the schools offer demonstrative geometry in the 10th grade, about half provide first year algebra, and about a third of them offer general mathematics. As one might guess, more schools offer these three courses than any other combination in the 10th grade. Incidentally, no school offers a review or remedial course in the 10th grade, although it is common to provide such a course in the 9th and 12th grades of these schools.

The last question of Part I read as follows: "Would you welcome the publication of a pamphlet providing guidance to your pupils as to the mathematics needed in various vocations and professions?" Only 1% failed to respond and all the rest said "yes." So it is altogether clear that teachers of mathematics want a guidance pamphlet. The Commission on Post-War Plans has, it is to be hoped, provided much of the material that teachers want along this line in a report in the November 1947 issue of THE MATHEMATICS TEACHER.[2] The purpose is to describe for high school pupils some of the occupations in which mathematics is important. The student learns about the amount and kind of mathematics that is used and the school level at which it should be taken. Reprints should prove very useful to guidance personnel, in classifying students at the ninth grade level. Here teachers of mathematics can find information that they can give to pupils and parents in a long time program of guidance prior to the ninth grade. The hope, of course, is that the choice of the proper mathematics courses in doubtful situations will be made by pupils and parents in terms of opportunity, interest, and ability. Too often, pupils and parents are crushed by official decisions which, to them, seem to slam the door on opportunity. As a result there is great pressure on teachers to permit students to take courses for which they do not have adequate preparation.

[2] A reprint of this report called "A Guidance Pamphlet in Mathematics" may now be secured for 25¢ each postpaid. In lots of 10 or more these pamphlets may be had for 10¢ each postpaid from THE MATHEMATICS TEACHER, 525 W. 120th St., New York 27, N. Y

PART II

Since the data collected by the first question have been used in the preceding section, no further comment is needed.

Item 2 reads: "Check the factors which cut down your efficiency as a mathematics teacher." The fourteen items to be checked have been arranged in the following table in the order of the frequency in which they are checked. (Many teachers checked more than one item, therefore the sum of the per cents is more than 100.)

	Per Cent
Extracurricular duties	41
Large classes (state average size)	35
Too many classes per week (state recitations per week)	32
Have to work at an out-of-school job to supplement income	30
Lack of equipment (rulers, compasses, protractors, bulletin boards, etc.)	22
Lack of supplementary materials	21
No classroom that I can call my own	21
Too many preparations	17
Required to teach unrelated courses	8
Other teachers in department without proper training	3
Worry about moderate income	3
Outdated textbooks	2
Lack of right kind of supervision	0.6
Inadequate training for my job	0.6

The five factors which are checked most often will not seem surprising to experienced teachers. These are involved in about a dozen things that need to be changed to improve the working conditions of teachers, not only that competent teachers may be efficient, but also to attract and to hold the kind of mathematics teachers that this country needs.[3]

[3] "Evolving Bill of Rights for Teachers." Swords Into Ploughshares, p. 37. Superintendent of Public Instruction, Lansing, Michigan, 1946.

Apparently teachers of mathematics made no significant distinction between items 3 and 4. Item 3 read as follows: "As regards your present efficiency, and what you might do in an ideal situation, what per cent do you think you are efficient?" Item 4 read as follows: "Considering your total program in mathematics in relation to your whole school, what do you think is its efficiency rating in per cent?" 15% of the teachers say they are excellent, 54% good, 17% fair, 1% poor, and 13% do not know. It is interesting, I think,—perhaps even startling —that 13% should say "do not know." As regards the 1% who say they are poor, even if we take this with a grain of salt, it is still very palatable.

Item 5 bears on the guidance problem and concerns the early identification and proper culture of youth with a special talent for mathematics. Teachers were asked to select their ten best students, interview them and report the number who intend to be teachers in a college or university, and the number who plan to be research workers in government or industry. Of the 350 students who were interviewed, only 7% plan to be high school teachers of mathematics, 1% are looking forward to teaching mathematics in colleges and universities, and 13% plan to do research work in government or industry. It is shocking to note that only 7% of the best students are planning to be teachers in high school, especially when we consider the high probability that many of this small number will probably be lured by higher salaries and better working conditions to industry or college teaching. It would seem that the National Council might well appoint a committee to undertake a more comprehensive study of this problem. It was hoped that the reports might include an analysis of the reasons for the choices of these young people. Unfortunately the data are so disorganized that no interpretation is possible.

Item 6 reads as follows: "How good a job would you say most high school mathematics teachers in your school are doing in providing young people with an intelligent basis for choosing mathematics as a profession?" In response to this question, 15% say they are doing an excellent job, 42% good, 30% fair, 5% poor, and 8% do not know. With the publication of the Guidance Pamphlet and especially if boards of education are encouraged to buy reprints in the manner that they buy other guidance materials, it is reasonable to expect that mathematics teachers will be able to do a better job in the future.

Item 7 reads as follows: "How about giving your young people the training they need for successful college work in mathematics? Are most of the classes in the traditional sequential courses doing an excellent, good, only fair, or poor job?" In response to this question, 30% say they are excellent, 47% good, 17% fair, 2% poor, and 4% do not know. The general picture is that about four out of five teachers think they are doing a good or excellent job in preparing students for college.

Consider next the last item in Part II which reads as follows: "And how about giving young people an understanding of the role of mathematics in promoting general human welfare? The Second Report[4] of the Commission on Post-War Plans includes a brief list that all pupils should learn if these things can be taught to them. Are the mathematics classes on the 'second track' (general mathematics) which aim to teach the mathematical know-how for better living doing an excellent, good, only fair, or poor job?" In response to this question, 13% say they are excellent, 40% good, 31% fair, 6% poor, and 10% do not know. Here the picture is that a little over half say they are doing an excellent or good job; this is considerably less than the number who say they are doing a good job in preparing

[4] "The Second Report of the Commission on Post-War Plans," THE MATHEMATICS TEACHER, May, 1944, p. 206.

students for college. The fact of course is that teachers of mathematics, as is the case with teachers in all high school subjects, face a dual responsibility. We must provide an appropriate culture for talent —preparation for leadership—and we must teach the fundamental competencies for better living in the ordinary affairs of life. This is not an either/or proposition. Both jobs must be done at a high level.

Part III

Again we must point out that the data collected by item 1 have been used in an earlier part of this report. Hence we can proceed to item 2. The first part reads: "Do you provide any form of refresher course in the 11th and 12th grades?" In general only one out of five schools now offers a refresher course in the 11th and 12th grades. Some schools report that they have offered such a course for two or three years but have abandoned it. All of them are offered for credit and with one exception for one semester. One school offers a refresher course throughout the year. Most of the courses seem to be review courses in arithmetic, although several emphasize special applications and teach such topics as logarithms and slide rules. It is perhaps obvious that this item of the Inquiry did not yield very useful information.

In item 3 teachers of mathematics were asked about the attitude of teachers and pupils toward general mathematics. A very small per cent of the teachers like to teach general mathematics. The rest of the group divided about equally between "dislike" and "indifference." In spite of this attitude, many added the comment "but general mathematics is extremely important." As concerns the pupils, the picture, amazingly enough, is a good deal brighter. Again and again a school will report that teachers are indifferent or dislike to teach general mathematics, but that pupils like it or, at the worst, they are indifferent. The fact is that the general courses in both mathematics and science tend to be stigmatized.[5] This need not be for there are school systems in which general mathematics is one of the popular courses of the curriculum. The Second Report of the Commission on Post-War Plans[6] listed reasons why the general courses tend to be stigmatized of which the chief one, as these data also suggest, is the attitude of the teachers. The fact that enrollments in the general courses are extremely large, and growing rapidly, confronts teachers not only with a serious responsibility but also with a grand opportunity. Every department of mathematics in a large high school needs at least one teacher who sees the importance of general mathematics in the public welfare and in the lives of people and is willing to evangelize for it.

Item 4 also did not yield helpful information. In the first place, it provided data to support a fact developed earlier that about half of the schools provide a general mathematics course which parallels 9th grade algebra. In the second place, it also showed that some schools are offering two years of general mathematics to parallel the 9th and 10th grades. Furthermore, a considerable number, let us say about $\frac{1}{3}$ of the teachers of mathematics not now offering two years of general mathematics, think they should do so, and all except one teacher not now offering general mathematics in the 9th grade, are convinced that such a course should be given.

In the next item it was learned that about half of the schools reporting were offering shop mathematics. In two out of three schools this course was taught by shop teachers.

Item 6 shows that 9 out of 10 teachers had read the pamphlet *The Role of Mathematics in Consumer Education*. This is evidence that the group reporting is not a representative sample of mathematics teachers, but is perhaps selective because they are readers of The Mathematics

[5] *Ibid.*
[6] *Ibid.*

TEACHER. Less than $\frac{1}{4}$ of the schools now provide a course in mathematics for the consumer as an elective for students in the 11th and 12th grades. However, two out of three think that their schools should provide such a course.

The responses to number 7 indicate that two out of three schools provide a course in commercial arithmetic, and in one out of three schools only is such a course taught in the mathematics department. The number of pupils that take business mathematics varies from 10 to 20 per cent of the enrollment.

Responses to item 8 show that only half of the schools reporting offered a course during the war years for the 11th and 12th grade pupils called *pre-induction* or *basic* mathematics. This is surprising for the total enrollment in such a course during the war years was very high—perhaps 400,000 students. Fewer schools are today providing such a course and many teachers say that interest in this type of course is decreasing. Only a few mathematics teachers think that schools should now provide such a course. It is the writer's conviction that a course along the lines outlined for pre-induction has a place in the late years of the senior high school for students who mature slowly and who come to realize that they have "missed the boat" and therefore want all the mathematics they can get in their last year of high school. However, this is merely a personal opinion and certainly does not stem from the data collected by the inquiry.

The response to item 9 "Do you think that your school should provide a rigorous course in arithmetic as an elective for 11th and 12th grade students?" is truly surprising in that nearly every teacher of mathematics taking a part in this inquiry checked this item with a "yes."

Item 10 provided responses showing that only a few schools give any kind of standardized test as a part of the requirements for graduation. A few give tests at the 9th grade level for guidance purposes, and several teachers say "it is a good idea."

The responses to item 11 "Does your school provide a general course in mathematics on the second track not suggested by the above questions?" were few. The courses most frequently listed were applied mathematics and practical mathematics. One teacher reports, "We used to teach social mathematics but we had to change the name to practical mathematics for the reason that a member of the school board was of the opinion that we were teaching socialism.

Item 12 invited questions that teachers would like to have published in future sections of such an Inquiry. There were numerous suggestions—some very good—to be used if and when the Inquiry is resumed. Since, however, the Commission has ceased to exist, it seems unlikely that the Inquiry will be continued.

National Council Yearbook!

The National Council Yearbooks are rapidly going out of print. Teachers who wish complete files and, particularly, school libraries who lack certain books should order now.—EDITOR.

Is General Mathematics in the College on Its Way Out?

By KENNETH E. BROWN
Wagner College, Staten Island, N. Y.

IN 1942 and again in 1947, questionnaires were sent to nearly five hundred colleges stating in their catalogues that they offered a course in General Mathematics.

Examination and comparison of these questionnaires indicate that General Mathematics is in a very unstable position. The chief causes of this precarious status are (a) an inadequate supply of teachers prepared to give instruction in the subject and interested in teaching it, and (b) the lack of a satisfactory textbook. That there is widespread consideration of General Mathematics is evidenced by the many comments and letters which accompanied the answers to the questionnaires. Teachers expressed freely their opinions concerning the shortcomings and the advantages of the subject.

GENERAL MATHEMATICS DEFINED

Before the status of General Mathematics is discussed, it would be well to recall that textbooks in the subject have been classified according to their objectives as follows:

1. Preparatory: to equip students for a profession, semi-profession, or vocation in which mathematics is a useful tool.

2. Cultural: to prepare students to be intelligent citizens mathematically, to transmit those skills and concepts that are the necessary prerequisites of a desirable citizen, to emphasize the understanding of mathematical concepts, and to show the relation of mathematics to the other great fields of learning—all with little emphasis on the manipulative process in problem solving.

3. Combined (Preparatory and Cultural): to attain both the above objectives by meeting the needs of the large academic terminal mathematics group, and likewise furnishing adequate preparation for the minority who wish to pursue further courses in mathematics. (See "What is General Mathematics?"—THE MATHEMATICS TEACHER, Nov. 1946, pp. 249 seq.)

THE QUESTIONNAIRE OF 1942

In 1942, questionnaires concerning general mathematics were sent to 492 colleges, and 457 of these questionnaires were returned.

According to the responses, the cultural type of general mathematics course had only recently made its way into the curriculum. Nearly half of the colleges answering the questionnaire had offered cultural general mathematics for a period of less than four years. It was concluded in the report that although cultural general mathematics was in an unstable position —considering the number of schools adding the course and those dropping it— there seemed to be a trend toward general mathematics for the terminal mathematics student. It also was pointed out in the report that "although it may seem desirable that this trend of cultural mathematics for the terminal student should increase—many instructors have issued the warning that the national crisis may cause students to elect the traditional courses. (See "General Mathematics in American Colleges," by Kenneth E. Brown, Bureau of Publications, Teachers College, Columbia University, p. 155).

THE QUESTIONNAIRE OF 1947

In 1947, a similar questionnaire was sent to 480 colleges whose catalogues indicated that they offered a course in general mathematics. The questionnaire emphasized that the survey was concerning gen-

eral mathematics for the terminal mathematics student and not a general mathematics course designed for those students majoring in mathematics or its applied fields; thus, of the 333 answers returned, one-third stated that a course in general mathematics was not and had not been offered in that institution for the terminal mathematics student.

Forty Per Cent Discontinue General Mathematics

More than two hundred schools reported that they had or are now offering a course in cultural general mathematics; however, only sixty per cent of them are now offering such a course. Thus, of those responding, forty per cent who have attempted to give a course in cultural general mathematics have discontinued the course.

The question immediately arises, "Why have these schools dropped General Mathematics?"

Only Thirty Per Cent Satisfied with Textbooks

Before drawing a conclusion, perhaps it would be well to examine the responses from the colleges that are still offering a course in cultural general mathematics. Here we find definite comments of dissatisfaction concerning the course. In fact, nearly one-half (exactly 45%) of those who are now teaching general mathematics indicated that the present textbook they are using is unsatisfactory and over one-third of the remainder were only partially satisfied with the textbook. Of the colleges who are now offering a course in general mathematics for the large group of terminal mathematics students, only thirty per cent report that the textbook now in use is satisfactory.

Reasons for Dissatisfaction with General Mathematics

Since the questionnaire study indicated that, of the schools which have offered cultural general mathematics, forty per cent have dropped it and nearly half of those who continue to offer it are dissatisfied with it, shall we conclude that general mathematics for the terminal mathematics student is a failure?

Before adopting a conclusion, we should observe the reasons given for dropping the course. The reasons, according to the frequency with which they were given, are as follows:

1. The textbook was unsatisfactory.
2. Because of increased enrollments in college since the war, the overloaded mathematics faculty could not offer both the cultural course and those courses designed for the specialist in mathematics.
3. During the war, the college enrollment dropped, and the students' interests were turned toward traditional mathematics.
4. Students could change to a mathematics major without loss of credit if the traditional courses were offered.

Although the large increase in enrollment without a proportional increase in the mathematics faculty was given by many colleges as the reason for discontinuing courses for the students who are non-mathematics majors, the most frequent reason was the lack of a satisfactory textbook. Many questionnaires contained a comment indicating that they believed the idea of a general mathematics course for the general student was a good one, but they expressed regret that they had not found a satisfactory textbook.

Textbooks Too Difficult

The next question that we logically may ask is, "Why is the textbook unsatisfactory? What improvement is needed in a textbook designed for a cultural general mathematics course? According to the questionnaires, the most frequent objection to the textbook was that it is too difficult. One instructor expressed it in these words: "Too difficult—I have been unable to find a satisfactory textbook for

this course. I need a suitable textbook for a short, simplified course. I have examined several texts and find them for the most part too difficult and far too long."

Textbooks Lack Meaningful Applications

Many instructors find that the textbooks do not contain applications which are meaningful to the students. Both by training and temperment the students are unable to readily see the beauty or value of mathematics in the applications given. The applications are so far beyond the students' experience that they are more difficult than the mathematical principle being taught. One instructor expressed it tersely: "We need applications that have meaning." Another expressed the idea more verbosely when he wrote the following comment on the questionnaire: "Does this mean that you are preparing a text? If so, make it real meaningful. Not just a fusion of abstract mathematics. What we need is a book that lives. We need a book with applications that the student can understand and will be of interest to him."

Textbooks Present Too Little About Too Much

Others object to the brief treatment of many important but difficult concepts. It seems to be their opinion that not enough material is given on one important concept before another concept is presented. They contend that the textbook deteriorates into a survey of the skeltons of mathematical principles, but the student lacks the background or temperment to add the flesh and blood to give the beauty of understanding. The following comment is typical of the comments expressing this idea: "This textbook is mathematical soup —everything from arithmetic to the calculus, but the students don't get enough of any one thing to understand any of it. They just have not had enough mathematics to know what it is all about." A great deal of the material is also really high school work.

Texts Are Uninteresting to the Students

Another frequent complaint is that the textbook is not interesting to the student —that many of the wordy discussions concern topics in which the student has no interest, and that they are not presented in a manner to gain the interest of the student who is untrained in logical thinking.

On one of the questionnaires was the comment, "I would welcome a text interesting to the students." In contrast to this statement, the Head of the Department of Mathematics in a large University recently said, "I don't give a d— what the students like. I know the mathematics they will need and that is the mathematics that they are going to get."

Importance of the Student Interest

It might be well to consider the interests of the student even from a selfish standpoint. If a course is offered in cultural general mathematics, and the student has solved every exercise correctly, but during the course the student has learned to hate mathematics, what have we or the cause of mathematics gained? In only a few seemingly-short years, the students who now sit in cultural general mathematics classes will be among the influential citizens of our city, state and nation. They will not be mathematicians, but they will be either friends or enemies of mathematics. Will they have gotten enough of a glimpse of the value of mathematics to recognize it as one of the great fields of learning—to know that it is indispensable to the progress of our civilization—or will they have learned to hate mathematics and see in it neither its beauty nor its value?

More advertising is being done for mathematics in the cultural general mathematics classroom than in the traditional class, but the publicity may not always be helpful to mathematics. The traditional mathematics class is composed of those students who are majoring in mathematics

or its allied fields. They will study enough mathematics to see some of its value, but this is not the case in the cultural general mathematics class. The cultural general mathematics students will have only a short glimpse of mathematics. Will they see some of its beauty and value? During this brief view of mathematics, some students will see mathematics as the necessary equipment of every educated man, while others will close their textbooks on the confused maze of symbols hoping that they will never see them again. Hence, they will go out advertising mathematics adversely, some in homes telling their children to stay from the "demon" mathematics, others becoming professors of education imbuing the prospective elementary teacher with hatred toward mathematics which in turn is passed on to the children in the lowest grades. Still others may go to their State Legislature and in mathematical ignorance attempt to change the value of *pi*.

We do not advocate that a course of study should be built on the desires of the student but we contend that more effective learning takes place when that interest is secured. The interest of the child means the interest of the man of tomorrow, whose interest is necessary for the continued progress of mathematics.

I repeat, if we continue to teach the students of today to hate mathematics, then mathematics will not have the support of the non-mathematician of tomorrow. Without the support of the leaders in other great fields of learning, mathematics education will be retarded.

Teachers Selection of Textbooks

In fairness to the authors of the cultural type of textbook, it should be pointed out that some of the teachers who are teaching a course designed for the terminal student are using a preparatory or cultural-preparatory textbook. It is to be expected that these teachers will have difficulty in maintaining student interest and meeting the objectives of the course.

In some cases, the teacher has stated clearly the objectives of the course, but he has selected a textbook that was not designed to meet such objectives. Miracles cannot be expected from the textbook. On the other hand, many of the teachers who are using the cultural type of textbook for the terminal student are dissatisfied.

SUMMARY

Answering the original question, "Is general mathematics on the way out?" It appears that general mathematics has lost ground for two reasons:

1. Lack of proper teaching staff in the face of increased enrollments.
2. Lack of a desirable textbook.

The first cause may correct itself in a few years when the influx of G.I.'s has subsided, and more and better teachers are available. Can the second cause be eliminated? The authors of the cultural type of textbooks must answer this question. Will they be able to find applications of mathematical concepts that will be meaningful to the student? Can they pick out a smaller number of mathematical principles and by means of these show the value and beauty of mathematics in a presentation that does not degenerate into a verbose discussion of mathematical theory? Can they give a continuity to these ideas that will be appealing to the student?

An examination of the cultural type of textbooks reveals that much progress has been made along these lines but the responses to the questionnaires indicate that much more progress must be made, or cultural general mathematics will take a mediocre place among the subjects of our college curriculum.

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Mathematics an Essential of Culture

By WALTER H. CARNAHAN
Mathematics Editor, D. C. Heath and Co., Boston, Mass.

WHAT is culture? Is it a matter of how one holds his cigarette, or how one holds a lady on a dance floor, or how one holds his liquor? Is it a matter of which fork one uses at the dinner table, or which tie he wears with a tuxedo, or to which book club one belongs? Is it a matter of ideals, or philosophy, or subjects which one is interested in and can discuss intelligently? I have purposely refrained from looking up the meaning of the word in making preparation for this discussion for the definition has little to do with this presentation. I am here interested in but one phase of culture, the phase that relates to the intellectual interests of men, their participation in thinking and talking intelligently about those matters that affect their lives forcefully and intimately. The cultured man should, I think, be able to discuss religion with a minimum of cant and emotion, should be able to discuss business and economics with a minimum of consideration of one's own financial gain or loss, and should be able to discuss science and mathematics as readily and intelligently as any of the other subjects that affect the lives of men intimately and continuously.

Of course, I am not here speaking of the research scholar's mathematics and the language in which he discusses it, differential equations, the special nomenclature, subscripts and superscripts, the logical and complete developments of the text books and treatises. I am speaking rather of the mathematics by which one understands and describes phenomena of the physical world, the mathematics of crystal formations, of flower and leaf arrangement and form, of shells, of trees, the mathematics of airplane design, the mathematics of atomic energy. And not all of the mathematics involved in any of these phenomena, of course, but the mathematics that is part of the most elementary understanding of them, the mathematics that on the whole one would find it easier to know a little about than to avoid, the mathematics that gives much satisfaction with very little pain. I hope to be able to make this clear with some illustrations.

Twice during the past year I have seen national popular magazines with the equation $E = mc^2$ in color on the front cover. In one case the equation appeared with a picture of Albert Einstein. A number of times I brought this cover to the attention of friends, not as a deliberate test of the culture of any one as an individual, but as a more general test of the existence of mathematical culture. No one on whom I made the test had the slightest idea what the equation was about, and only one seemed to have heard of Einstein. This person had heard of "that queer duck who invented relativity or something." Yet these persons were people of some culture; they could have discussed the art of Goya, the music of Chopin, or the novels of Sinclair Lewis.

Many persons, old and young, are interested in the two hundred-inch telescope being constructed on Mount Palomar in California. Frequently I have tried to discuss this as an application of mathematics, particularly the paraboloidal surface of the giant mirror. Have you tried this on some of your layman friends? Few of them seem to have heard of a parabola, and I am inclined to think that some of them think only human caprice and not physical law decides the form of the surface. Who is at fault, or is there any fault, that general culture so often ends where mathematics begins?

Historians, novelists and other writers are in part responsible for our lack of

mathematical culture. Certainly no historian writing about Europe would think of omitting reference to Napoleon, but very few have a word to say about LaPlace or Guass. Alexander occupies many pages in the History of Greece, but Euclid is dismissed with three lines or none at all. Henry VIII had six wives, as every history of England tells us, but how many histories tell about Isaac Newton and Principia? The Romans captured Syracuse in 212 B.C., as every ancient history informs us, but where is the history that tells about Archimedes, the greatest glory of the fallen city and greater and far, far better than Marcellus and all his greedy bloody Roman bosses? Dickens in *Tale of Two Cities* counts the heads of citizens as they fall under the guillotine, and he even tells about the women who knit sox in the shadow of the gory instrument, but where does he tell about LaPlace and *Mecanique Celeste*?

In the pages of history, literature and magazines there are the names of many persons whom the layman usually knows only in non-mathematical connections but who also have rightful places in the pages of mathematical history concerning which educated people should be informed. I wish to illustrate this statement at some length as justification of my thesis that mathematics is one of the essentials of culture. Much of the next few pages of this paper is familiar material to my present readers, and you will be justified in giving only half attention.

Napoleon. When the name of the Little Corsican is mentioned the minds of nearly all people turn to his conquests, his remaking of the map of Europe, Marengo, the Battle of The Pyramids, Austerlitz, Jena, Elba, Waterloo, St. Helena, the humiliation of the last years of his life. And in fact there is but little else about which one can think in connection with his life. But "the little else" gives an opening for us who are interested in mathematics. There is a problem which is sometimes called Napoleon's problem, not because he originated or solved it, but because he was fond of trying it out on his engineers. It is to construct the vertices of a square using compasses only. It is but a small part of the subject of compass geometry which was systematically developed by Mascheroni of Italy to whom should be given credit for the problem and for its solution. It presents very little difficulty, and your best high school geometry students should be able to solve it. When they have done so, they should have learned that compass geometry exists and is a very interesting subject. They should also be able, the remainder of their lives, to converse about Napoleon at the dinner table without exclusive attention to his violence. They should even be able to quote the presumably sincere words of the Emperor: "The advance and perfecting of mathematics are closely joined to the prosperity of the nation."

Benjamin Franklin. He walked the streets of Philadelphia with a loaf of bread under his arm; he became a wealthy printer; he proved that lightning is an electrical phenomenon; he wrote Poor *Richard's Almanac;* he rendered great service to America as our representative in France; and he had quite a way with the ladies. But Benjamin Franklin also had a deep interest in mathematics and more than once praised it in unrestrained language. Said he, "What science can there be more noble, more excellent, more useful for men, more admirably high and demonstrative than mathematics?" When he said this, Franklin was not merely paying the homage of an admiring observer. Mathematics books still preserve a very unique magic square which he made and call it in his honor Franklin's square. It might well decorate the walls of your classrooms.

Abraham Lincoln. He split rails, wrestled, told funny stories, freed the slaves, and is universally regarded as one of the half dozen truly great men of the last thousand years although no one yet understands why. We do understand that

he was driven by some compulsion, some inward urge, some destiny that took charge of his life and impelled him mercilessly toward his calvary. He followed his road almost blindly, seeing but a very little way ahead and that none too clearly, but recognizing the proper turns with an unerring sense that was like the homing instinct of a wild fowl in the spring. One of these important turns was a mathematical one. He was trying to learn to be a lawyer but one of the essentials was escaping him; he could not learn to prove his points beyond the possibility of refutation. He needed a system of reasoning that was so correct that there would be no standing against it. At last he found it. Let him tell it. "I left my situation in Springfield, went home to my father's house, and stayed there until I could give any proposition in the six books of Euclid at Sight. I then found out what 'demonstrate' means, and went back to my law studies." I think that experience began his preparation for The Gettysburg Address, the greatest oration of all time.

James A. Garfield. He was a great and good man who had part of his education sitting on a log beside Mark Hopkins, and he was assassinated. When politics claimed him, mathematics lost a capable and original worker. There are more than three hundred published proofs of the Pythagorean Theorem, and among these is one contributed by Garfield. Many text books in geometry still give Garfield's proof, and so a small part of his immortality is mathematical, and a part of one's cultural training should be some knowledge of that fact.

Florence Nightingale. She is justly honored as the founder of the Red Cross, one of those persons who go about the earth trying to minimize and repair the damage done by the killers and destroyers. She had to contend with the stupidity of British authorities almost single handed, and single handed she won her fight for the privilege of binding up wounds and introducing a spark of human kindness into the lives of broken men. None of us will regret that she gave up a career in research mathematics to do the work for which she will forever be honored, but we can feel great pride in the fact that it was one of us who founded the Red Cross. Florence Nightingale was a private student under Sylvester doing research in analysis when the Crimean War broke out. Was it the contrast between the constructive values of mathematics and the futilities of war that drove her to the fields of battle as an angel of mercy? Let us have the satisfaction of believing it was so and telling others of our belief.

Omar Khayyam. The Arab poet sang of "a loaf of bread, a jug of wine and thou beside me singing in the wilderness," and thousands of persons know about this and other of his saccherine verses. But Khayyam was a great mathematician, and mathematicians should lose no opportunity of claiming him for their own. He was one of the great Arab mathematicians at the height of the cultural glory of Arabia and made a number of contributions to the development of algebra. His mathematics was a contribution to world culture neither more nor less important than his poetry.

Lewis Carroll. He wrote of Alice, the Mad Hatter, The March Hare, the Dormouse, of a world that spins so fast one has to run to stand still, of a dream world where animals and persons live as equals. But Lewis Carroll is a mathematician by profession and by pride, university instructor and author of scholarly treatises. A queen once asked Carroll for an autographed copy of his book, expecting to receive *Alice in Wonderland.* She received instead an autographed copy of his latest textbook in mathematics. Magazines have offered fortunes for sequels to Alice, but Carroll ignores the offers. There must be something profoundly right with mathematics when a man equally gifted as novelist and as mathematician chooses mathematics and obscurity rather than fiction and growing fame. Let us tell our pupils that.

Descartes. Once I went into a large public library and asked for and obtained permission to visit the stack rooms. In browsing among the shelves on philosophy, I came across numerous books by one Rene Descartes. Not one of these had anything to do with mathematics. They were books on Cartesian philosophy, and I presume that one who is well read in philosophy would know much about these works of the man who spent nearly all his life in bed merely because he liked bed better than a table as a place for work. We should not object to the philosophers claiming him as one of their number, I think, but let us insist that it is only a joint claim, because he is one of the great mathematicians who made a truly epochal contribution to the subject when he gave us analytic geometry. His geometry was written when he was up in years, after he had devoted his mind to a prolonged effort to arrive at the meanings of life by abstract philosophy. He longed for something more definite and sure than the beliefs at which one arrives as the result of observing human behavior and trying to rationalize it. And so he turned definitely to mathematics for conclusions that are incontestable—or as nearly so as any of which man is capable. This experience of Descartes may hold a suggestion for other earnest persons longing for some incontestable truths with which to season the uncertainties of their lives.

Princess Elizabeth. Three hundred years ago, the Sunday Supplements, if they had existed at that time, would have had pictures of Princess Elizabeth of Holland. In the first place, being a princess would have been quite enough, but, in the second place, she was intellectually gifted. After these three hundred years, the fact that she was a princess no longer entitles her to remembrance. The only space in literature to which she is entitled (so far as I know) is accorded to her because she employed a then new and now well known branch of mathematics to solve an ancient and famous problem. The Problem of Apollonius is named after that worthy Greek because he proposed and solved it. It is to construct a circle tangent to each of three given circles. For hundreds of years this remained half a problem, being a problem without an answer. Apollonius solved it, but the book containing his solution was completely lost and no record has ever been found of his method of solution. In time various compass and straight edge solutions were offered but, at the time of Elizabeth, none of these was good. When Descartes invented the method of analytic geometry, one of those who was interested in it and mastered it was his private student, Princess Elizabeth of Holland. And the important problem on which she elected to try out the new power was the Problem of Apollonius to which she gave the first analytic solution. Tell this to those who like to talk about royalty.

This discussion of well known persons and their mathematical experiences could be extended indefinitely as exemplifying the statement that some knowledge of mathematics, at least from an historical point of view, should be expected of all persons of culture. However, there is another type of illustration of this statement to which I wish to invite your attention. Not only in discussion of persons is some knowledge of mathematics an essential of culture, but also in discussions of natural phenomena that are part of the daily experiences of men such knowledge is essential. Whatever else culture may be, it is an enrichment of the experiences of life, and he who cannot avail himself of the enrichment that mathematics makes possible is, to that extent, deficient in culture. I now offer a few illustrations of the enrichment of human experience with natural phenomena which a little mathematics makes possible.

The Shell Fish. At a sales counter in an aquarium I recently purchased a set of ear decorations made by fixing clamps to shells of small fish. When I showed these to some friends they exclaimed, "How pret-

ty! May I try them on?" Yes, go ahead and try them on, and while you do so let me tell you something interesting about them. The whorls in all these shells are mathematical forms known as logarithmic spirals. In the language of mathematics this spiral is described by the equation $\rho = a^{\theta}$, or $\log_a \rho = \theta$. Every shell fish of this species makes this mathematical figure, and yet he knows nothing about logarithms. How does this come about? Well, like all other growing things, the shell fish obeys the law of growth which can be simply if not too accurately stated like this: The larger a living organism is, the faster it grows while it is in the growing state. Mathematically stated, this law is precisely the equation of the logarithmic spiral. Hence it is that the shell fish, obeying the law of growth, makes the logarithmic spiral.

The Jet Plane. Over and over magazines are showing pictures of jet planes, and persons are speculating on the meaning in human experience of this new type of power. No one can wholly escape the fact that mathematics is a necessary consideration in every step of the construction of this plane, but there is one application of mathematics to which my mind turns every time I see a picture or hear a discussion of the jet plane. It is the mathematics of the pursuit curve, and the surprising result to which it leads when the pursuer is a motor driven plane and the pursued is the jet plane. I shall tell you a true story to illustrate this, but first I shall repeat a very old, familiar catch problem to refresh your minds as to the nature of the pursuit curve. The problem is this: A dog is in the southwest corner of a rectangular field and a rabbit in the northwest corner. They see each other at the same time. The rabbit starts running along the north fence and the dog starts after him, running faster than the rabbit and always toward the rabbit. Where and when will the dog catch the rabbit, the speeds being respectively 20 miles an hour and 15 miles an hour? As you know, this is not an easy problem to solve if one undertakes to set up the equations and solve them simultaneously. I may remark in passing that a child can solve it with needle and thread, as the Eighteenth Yearbook of the National Council of Teachers of Mathematics illustrates. The dog follows a pursuit curve, of course. When a fighter tries to bring down a bomber, it follows a pursuit curve since its guns are aimed straight ahead. When the bomber defends itself, it aims its guns, not directly at the fighter, but to one side, just as a hunter 'leads' a bird.

When the Germans began using jet fighters against our bombers in the summer of 1944, their greater speed changed the aiming problem. Most gunners thought they should increase the lead due to the greater speed. Mathematics again solved the problem and showed that the lead should be decreased.

Beads and Bridges: A lady dressing for dinner places a strand of beads about her neck. Nearly always the shapes of the beads are obviously mathematical, but mathematics also enters into the form in which the strand adjusts itself. This mathematics is not so obvious as that of the form of the beads because it is not so well known. It is the catenary curve, and the beauty of the curve is fully as important as the shape or color of the beads. The mathematics of the strand of beads is the same as that of the cables of the suspension bridge before the weight of steel beams and concrete is added. It is the same as the mathematics of power lines and clothes lines. Beads are worn because of their beauty, and a large part of the beauty is the beauty of mathematical forms.

Please Pass the Salt. Salt is a mineral crystal, and mineral crystals are mathematical in form. In the case of salt this form is one of the best known and simplest of the geometric solids. It is a cube. At least it was so when nature formed it. The battering and crushing which it later receives will soon change this, of course. Sugar, diamonds, snow and many other

materials with which we are familiar crystallize in mathematical form.

This discussion of the mathematics that is unescapable in human experience could be extended without limit, but any one of you could easily make such extensions at will.

I turn now to a brief consideration of the opportunity and obligation of the mathematics teacher to reveal to students the cultural aspects of mathematics and suggest some ways in which this revelation can be made. I do not forget, and would not tempt you to do so, that the chief purpose in teaching mathematics is to give learners a language and a tool which they understand and can use to master laws and put them to work doing the useful things of the world. But since these students are entitled to cultural enjoyment as well as profitable applications of the things they learn, we have the added responsibility of showing them the cultural aspects of our subject. There are certain specific ways in which we can meet this responsibility.

First, pupils should know that mathematics like literature and art has grown as the results of the contributions of many men in many lands. God may geometrize, as Plato is said to have asserted, but He does not reveal the results except through the labors of men. The story of these labors and contributions is as profitable as it is fascinating, and a little of it should be part of the mathematics education of every one. This means that teachers should make much more use of biographical materials than is now done.

Second, there is mathematics that is not part of the text sequence that is fully as interesting as that in the books, and pupils should know something about this. It may be asked, since it is important, why is it not in the books? The answer is simple; of the many things that are of interest to teachers and pupils, no one can forsee just what is going to interest your class nor any individual in it. No text could be expected to give a treatment of relativity, but some day John may want to know something about this subject. It would be an error to tell him to wait until he is older, or until he is in college, or until he can read the books. He is entitled to know now something about this, either in class discussion, or in mathematics club, or in private talk with the teacher. He will not understand much; neither do I; but the little he can be told and can understand may be very important in his intellectual experience. No text could be expected to treat non-Euclidean geometry, but some pupil in your geometry class may raise questions in connection with the parallel postulate that will lead to some consideration of this subject. Some discussion is in order as a contribution to his cultural and intellectual expansion.

Third, the mathematics of common things, plants, animals, crystals, shells, chains, falling objects, soap bubbles should be part of the learning experience. Fortunately many books now make definite provision for making pupils conscious of these manifestations of mathematics in nature. The error that the teacher is likely to make is in assuming that a picture in the book is sufficient treatment of the subject. Occasionally a pupil should be given an assignment to find out more about the mathematics in some natural manifestation and tell other members of the class about this.

Fourth, the mathematics which you and I do just for the fun of it is fully as interesting to pupils as to teachers, and an occasional digression into this field is well worth the time, even if it is class time. The mathematics club can cultivate interest in this type of mathematics on the part of those who have the preliminary interest to join and attent club meetings, but those who have not this consistent interest can profit from an occasional five minutes spent in this field.

A Three-Fold Purpose of Mathematics and How to Attain It

By TRUMAN L. KOEHLER
Muhlenberg College, Allentown, Pa.

IN GENERAL, the purpose of education is three fold. First, it aims to give young men and women the sort of training which will enable them to become self-supporting. It makes them better able to earn their own living. This is a fundamental purpose because earning a living is one of the most insistent problems that most people have to solve. Until men and women are able to handle this problem, they cannot solve any others. This economic objective of education is steadily increasing its importance, because of the high specialization which is being developed in all fields.

Helping people to earn a living is not the sole purpose of education, and an educational system which confined itself to this alone would be seriously defective. A second objective in education is to develop the personality of the individual. It is the function of the school to search out the native abilities and inclinations of each pupil so that these qualities may be trained and developed. Personal aptitudes which are not directly connected with the work of earning a living may nevertheless be worth developing because the durable satisfactions of life are not entirely dependent on the total income earned.

The third purpose of education, the social purpose, is presently regarded as the most important of the three. Above all else, public education has come to be looked upon as a process of training in good citizenship. In democratic communities, such training is of vital importance because democracy rests upon the good sense and tolerance of the people. It is the social purpose of education to train men to think before they speak or act, to remember that there are two sides to every question—hence to approach public issues with thoughtfulness and discrimination.

While this three-fold purpose applies to education in general, it is uniquely applicable to mathematics. Mathematics teachers should mutually subscribe to the following aims and objectives:

1. To teach mathematics so it will serve students as a tool in courses in chemistry, physics, economics, etc.—the economic objective.
2. To teach mathematics as an end in itself —Mathematics has always been studied by various men who were totaly oblivious and indifferent to its applications, and yet who, through its study, developed their personalities.
3. To teach each course so as to encourage consistent and logical thinking in the field and with a deliberate aim to transfer this kind of thinking to outside of school activities. To teach in such a way as to obtain fine discrimination in recognizing the factual elements, the suitability and consistency of hypotheses, and the adequacy of tests and checks.

My experience in college teaching for the past twenty years suggests that the entering freshman who was taught mathematical facts and skills with the appropriate supplemental concepts, is a superior student in mathematics.

Let us consider a number of illustrations. More than mere rule is indispensable to fix the law of multiplication of positive and negative numbers. The following concept should prove useful:

I. It is obvious that—

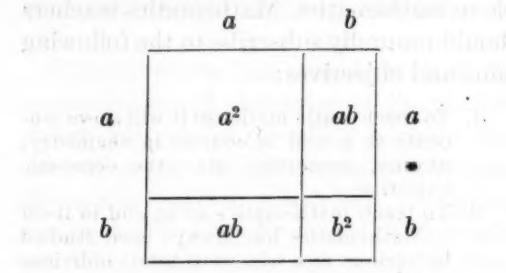
1. $(13)(17)=221$ If we write the factors in equivalent form and proceed to find the product, we obtain—

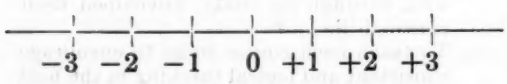
2. $(20-7)(20-3)=400-200\pm21$

Since the products of (1) and (2) must be identical, we must conclude that consistency demands that $(-7)(-3)$ must equal $(+21)$. Considering the areas of properly chosen rectangles yields another approach to this same concept.*

* A discussion of this may be found in "Elementary Mathematics from an Advanced Standpoint" by Felix Klein, Dover Publications, N. Y., p. 26.

II. Many students are likely to write: $(a+b)^2=a^2+b^2$. Constructing a square with $(a+b)$ on a side and considering its area should be useful here. Thus—the area of the square is $(a+b)^2=a^2+2ab+b^2$.





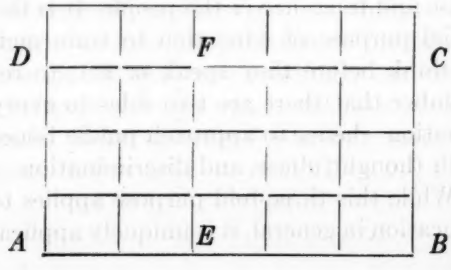
III. Signed numbers always require attention and a graphic exhibition is effective. Let us agree that $(+2)$ shall mean 2 units to the right of the origin. Let us agree that (-2) shall mean 2 units to the left of the origin. Hence $(+2)+(-3)$ shall mean 2 units to the right and then 3 units to the left. In another form, subscripts can be utilized and we may say $2_R+3_L=1_L$ or $(+2)+(-3)=-1$. Again, $(+2)-(-3)$ shall mean how many units to the right and then 3 units to the left to arrive at 2 units to the right. This interpretation becomes obvious when one considers $(a-b)$ to mean that an x is to be found so that $b+x=a$. So $(+2)-(-3)=+5$.

The answers (-1) and $(+5)$ to these two problems are associated with points on a line. There is a one to one correspondence between all real numbers and points on this line and this, in turn, serves as a fundamental concept in a geometry of one dimension. To every point on the line there corresponds a real number and conversely, with every real number there is associated a point on the line. In general, the term "dimension" is associated with the elements necessary to position an object. If an object can be positioned with one element, we have the basis of a one-dimensional geometry. In the real plane, two elements are essential to position any point, and we refer to this study as two-dimensional geometry. In the special relativity theory, four-dimensional geometry is indicated—the implication that four elements are requisite to position an object. At this moment, I may be in a certain position on the second floor of the Science Building. With respect to a set of axes, I may be at point $P_1(x_1, y_1, z_1)$. An individual on the campus may measure out the x_1, y_1, z_1 and yet not find me, for in the meantime I may have moved into another spot. The x_1, y_1, z_1 were inadequate to position me. Hence to avoid this dilemma again, the seeker and I will synchronize our watches and agree that at time t_1, I will be at P_1. It is now obvious that four elements were needed to position an object, and we are led to a discussion of a four-dimensional geometry.

Another concept, which belongs to the theory of point sets of Cantor, could be introduced. Let us think of all points with rational abscissas on the preceding line. The totality of these rational points on this line is "dense," implying that in any interval however small, there are still infinitely many rational points. Specifically between any two rational points, there is always another rational point.

It is obvious, therefore, that this approach of emphasizing the one to one correspondence between signed numbers and points on a line, not only establishes a reasonable interpretation of the addition and subtraction of signed numbers, but is a sound basis upon which to build subsequent and more advanced mathematical concepts.

IV. Every teacher feels impelled at times to give further instruction in fractions, and I believe the following scheme will be expedient:



Consider: 1. $\frac{2}{5}+\frac{3}{4}$

Let us assume a unit rectangle, and divide it into fourths and fifths as shown.

We can count 20 little rectangles, each of which is $\frac{1}{20}$ of the large rectangle.

By actual count, $\frac{3}{4}$ equals 15 small rectangles or $\frac{15}{20}$.

By actual count, $\frac{2}{5}$ equals 8 small rectangles or $\frac{8}{20}$.

Hence, we obtain a sum of 23 small rectangles or $\frac{23}{20}$.

2. $\frac{2}{5}$ of $\frac{3}{4}$

$ABCD$ represents $\frac{3}{4}$ of whole rectangle. $\frac{2}{5}$ of $ABCD$ equals $AEFD$ which obtains 6 small rectangles or $\frac{6}{20}=\frac{3}{10}$.

This concept should be very useful when we proceed to formulate the rules.

While the mechanics of fractions may be comprehended by the student, the conviction that they have any function outside of mere formal exercises may still be in the balance. Surely, then, at a suitable academic level, their usefulness can be exhibited to the student by the use of the musical scale.

It is remarkable that the ear of man has for centuries recognized that three notes are harmonious when their frequencies are as 4:5:6. This combination is called the major triad.

The major scale is a sequence of tones so related so that the 1st, 3rd, and 5th form a major triad.

Also, the frequency universally assigned to "C" in physics is 256. The frequencies of the other notes of the major diatonic scale may be found by the proportion—$C:E:G=4:5:6$ (Incidentally an example of a continued proportion).

C	D	E	F	G	A	B	C'
256	288	320	$341\frac{1}{3}$	384	$426\frac{2}{3}$	480	512
	(a)	(b)					
		(c)					

In the (a) interval $\frac{f_D}{f_C}=\frac{288}{256}=\frac{9}{8}$

In the (b) interval

$$\frac{f_E}{f_D}=\frac{320}{288}=\frac{10}{9}$$

In the (c) interval

$$\frac{f_E}{f_C}=\frac{f_E}{f_D}\cdot\frac{f_D}{f_C}=\frac{10}{8}\cdot\frac{9}{8}=\frac{5}{4}$$

Major diatonic scale

Hence, fractions arise which in this elementary way produce $\frac{7}{8}$ and $\frac{10}{9}$, the major and minor tones respectively, and $\frac{5}{4}$ a useful ratio between two non-consecutive notes.

In addition to emphasizing mathematical concepts, dangerous mathematical procedures demand statements of caution. In an equation in which the roots are compartmentalized, as $(x-a)(x-b)(x-c)=o$, the expansion into a cubic equation is an elementary process. As a matter of fact "n" roots may be compartmentalized as above, and an expansion leading to an equation of degree "n" is obtained. The roots have been "scrambled." Now a student may propose the converse, can an unscrambling process take place? Can one take an equation of any degree, and compartmentalize the roots? Here a word of caution must be introduced. The answer to this question was given in 1824 by Niels Henrick Abel, in a paper which he had to publish privately, in which he proved that it is impossible to solve the general equation of the fifth degree or higher in terms of radicals.

In a trigonometric identity, the suggestion is usually made, that one side should be transformed into the form of the other side which should be left unchanged.

As a matter of fact, both members of the identity will tolerate any fundamental process, except, and here the caution must be flashed, the extraction of roots. For example: $sin^2(-\alpha) = 1 - cos^2\alpha$ for any α—

Extracting roots—$sin(-\alpha) = \sqrt{1 - cos^2\alpha}$

$-sin\alpha = \sqrt{1 - cos^2\alpha}$

To discuss mathematics only in terms of its utilitarian value, is to ignore important characteristics of this subject such as developing creative ability. Creative ability develops in an individual only when his mind is as free as possible from restrictions. For example, many concepts and investigations in mathematics are considered without any conscious application to nature—an indication of the mathematicians' creative power. As evidence, we submit such concepts as transfinite numbers, limit, and continuity. Investigations on the conic sections were conducted wellnigh 2000 years before application, and complex numbers were studied about 150 years before they were used to describe in some strange way the characteristic features of alternating current. Surely the non-Euclidean geometries were first studied without any thought of the physical universe. For the last 50 years, mathematicians have been studying Topology or Analysis Situs. Topology may be partially described as the study of those properties of figures which remain invariant when the figure is deformed in any continuous way. Let us consider a rubber sphere which can be deformed by stretching and crumbling, but which may not tear or puncture. There is at least one property which will be preserved under this deformation. If we cut along any closed curve, the sphere will fall into two pieces. Again, for workers in this field, the thought of application is remote, but through this creativeness, personalities are developed. Dantzig, Tobias: Number, p. 232 says "the mathematician may be compared to a designer of garments who is utterly oblivious of the creatures whom his garments may fit. To be sure, his art originated in the necessity for clothing such creatures, but this was long ago; to this day a shape will occasionally appear which will fit into the garment as if the garment has been made for it. Then there is no end of surprise and delight."

These surprises which come to mathematicians do not produce the internal drive for further creative work, but rather, the internal drive is the consequence of an intuitive desire to use mathematics as a means of developing and manifesting their personalities. It is mathematics for mathematics' sake. Poincaré said that "People have been shocked by this formula, and yet it is as good as life for life's sake, if life is but misery."

The appeal which mathematics makes to one's esthetic sense through its symbols, must not be neglected. They serve as the medium of expression which in turn appeals to the mind. In like manner, paints and canvas serve as the media for expression for the painter, and sound for the musician. These media appeal to the eye and ear respectively. The mathematical symbols may appear to be artificial and an arbitrary legerdemain, but Heinrich Hertz has this to say, "One cannot escape the feeling that these mathematical formulae have an independent existence and an intelligence of their own, that they are wiser than we are, wiser even than their discoverers, that we get more out of them than was originally put into them." While the extreme formalism of the Peano-Russell School with its symbolic logic may impress one adversely, nevertheless, the symbols used in an elegant proof of a theorem is as appealing as the masterful use of words in a poem.

Geometrical symmetry as expressed in the circle, parabola, ellipse, and hyperbola requires no additional elaboration as to its esthetic appeal.

The ability to express generalities in mathematics makes a powerful appeal to our esthetic sense. If an angle is to be found so that arc $sin\ \frac{1}{2}$, we obtain

$$n\pi + \left(-1\right)^n \frac{\pi}{6}.$$

Again through generalizations we can speculate as follows:

$\frac{x}{1!}$ = length of a line

$\frac{xy}{2!}$ = area of a rt. Δ

$\frac{xyz}{3!}$ = vol. of a rt. tetrahedron

$\frac{xyz \cdot \cdot \cdot t}{n!}$ = vol. of solid in n dimensions

To discuss mathematics only in terms of its utilitarian value and its important esthetic characteristics is to ignore its logical structure.

Mathematics is a collection of mathematical systems. A system is an abstract process which is not influenced by physical objects except insofar as they may occasionally suggest what to prove.

Euclidean geometry, any non-Euclidean geometry, and algebra are examples of mathematical systems. Each system contains undefined elements, definitions, axioms concerning the undefined elements, and theorems proved about the undefined elements. Frequently the axioms selected for a mathematical system are suggested by some physical situation, as for example in Euclidean geometry. However, the creation of a non-Euclidean geometry has made mathematicians realize that physical experience need not control a mathematical system. Any system of axioms may be chosen from which theorems may be deduced. The physical interpretation is irrelevant.

Bertrand Russell states that mathematics is "the subject in which we never know what we are talking about, nor whether what we say is true." What Russell meant is that because at the basis of a mathematical system are undefined elements which have no physical meaning, mathematicians do not know what they are talking about. They never know whether what they are saying is true, since the conclusions are not assertions about physical objects. We can merely say that the conclusions are valid. Benjamin Peirce, a nineteenth century mathematician, stated that "mathematics is the science which draws necessary conclusions." This definition emphasizes the inescapable conclusions drawn from the axioms.

With these considerations in mind, a student's sense of reasoning should be quickened. Much of our thinking can hardly be dignified by the name of reasoning. Our thoughts are occupied with musings and recollections, flitting here and there, more or less disconnectedly. For example, "I must replace my broken ink bottle—The male chorus had a good program last night—they wore tuxedos and that reminds me that mine must be sent to the cleaner."

However, as I look at one of a certain group, I see that you are wearing a white ribbon, and I know that you are registered in the L.C.I., for all people registered in the L.C.I. wear that white ribbon. Here is an example of good deductive reasoning. The statements can be arranged as follows:

1. All people wearing a white ribbon are registered in the L.C.I.
2. This individual is wearing a white ribbon.
3. Therefore, this individual is registered in in the L.C.I.

A simple example in mathematics would be:

1. Two points determine a line
2. A and B are two distinct points
3. Therefore, the points A and B determine a line.

Statements, related as the three are related in each of the two preceding examples, form a syllogism. The mathematical style is the syllogistic style. We must have at least two premises from which an inescapable conclusion follows. The argument may be valid and yet the conclusion false; but if both premises are true in a perfect syllogism, the conclusion must be true. Sometimes in mathematics, hypotheses other than those stated in the proposition are needed to justify the conclusion. These additional hypotheses in

mathematics are spoken of as axioms and postulates. Euclid in his Elements listed ten assumptions, five of which he called common notions and the rest postulates.

As an example from algebra, if $x-2=7$, then $x=9$. We must use the additional hypothesis that permits the addition of 2 to both members. Two premises have now been provided and the conclusion followed.

Every proof of a theorem in geometry consists of a series of syllogisms. About eight syllogisms are required to prove the theorem, "the sum of the interior angles of a triangle is 180." Any course in geometry which does not give training in explicitly exhibiting the syllogisms requisite to prove a theorem, I consider as inadequate.

It is reasonably obvious that mathematics deals almost exclusively with premises and conclusions and deductive reasoning which is one of the important methods of drawing conclusions from premises. Clarity and precision of definitions and assumptions, and rigor in reasoning can be rather simply studied in mathematics and should be attained.

One must be taught not only to read and write, but to analyze and interpret what one has read, to look for and recognize premises and conclusions, and to discover the presuppositions which led to the particular choice of the premise used.

The enunciation of a theorem consists of two parts, the hypothesis or that which is assumed and the conclusion or that which is asserted to follow therefrom. In the typical theorem—if A, then B, the hypothesis is A, and the conclusion is B. As an interesting extension, let us consider a theorem and its transformation.

Theorem: If A, then B
Converse: If B, then A
Inverse: If non-A, then non-B
Contrapositive: If non-B, then non-A

It can readily be shown that the converse and inverse of a theorem are not necessarily valid inferences, but the contrapositive is always a valid inference. Hence the Law of Contraposition can be introduced as an axiom of logic. To write a theorem in geometry and its transformation serves as an excellent exercise for a student. New and interesting theorems will result which will be stimulating to any student.

The potency of much of our advertising is a function of loose reasoning. One frequently sees a "Man of distinction" pictured with whiskey X. The following statement and its transformations are accepted a priori by the public:

Statement: All men of distinction drink whiskey X
Converse: If you drink whiskey X, you are a man of distinction
Inverse: If you are not a man of distinction, you will not drink whiskey X
Contrapositive: If you do not drink whiskey X, you are not a man of distinction.

The use of the term "inverse" in this discussion should not be confused with its definition in algebra, where if $y=f(x)$, then $x=g(y)$ is said to be its inverse.

Surely we miss an important function in our teaching, if students fail to recognize what transformations of a theorem require proof to establish their validity. The Harvard Report after recognizing the tool value of mathematics, proceeds to say that "mathematics has an important intrinsic role in general education. The ability to analyze a concrete situation into its elements, to synthetize components into a related whole or to isolate and select relevant factors, defining them rigorously, meanwhile discarding the irrelevant and the ability to combine these factors often in novel ways so as to reach a solution, are important features of mathematical procedures."

In conclusion, the study of mathematics has been justified in the past with the promise and on the premise that it offers, an approach to mental discipline—a unique way of thinking. One doubts if the promise is fulfilled or that the premise is presently tenable.

The difficulty is found in purposes which are confused in the minds of teachers of mathematics. Departments of physics and chemistry in college insist upon the use of the calculus very early in

the student's college career, and as a consequence the mathematics department feels impelled to surrender any exercise in logical thinking, but rather recognizes the necessity of dispensing heavy doses of fundamental facts, formulae, and symbols. In this aspect, mathematics serves as a mere tool; and must be thought of as just one of a number of purposes which must be attained. The presentation of facts should be, as far as possible, on a conceptual basis, for it is this latter implementation that aids in giving a degree of permanence to the learned product. But more than this, satisfaction to teacher and student alike comes when mathematics is studied for mathematics sake—as a means of developing one's personality. Even at the elementary level, the student experiences a feeling of pleasure and satisfaction when he solves a problem requiring original thought. Further, we should contemplate its characteristics of freedom, truth, and ideals, which in turn may serve as an impetus to creativeness and a feeling of well-being through its esthetic appeal. This is a second purpose.

Recognizing its logical structure so that systems may arise which may or may not be in perfect consonance with reality is a part of the third aim.

I believe the three aims can and must be attained, otherwise an intelligent public may remove its sympathy which would retard the progress of science and society. It will not suffice for us to emphasize one aim at the exclusion of all others on the basis that fundamentals are not known and hence any progress in logic, and esthetic appeal would be just plain superficial. If one were to wait for a perfect background before taking a forward step, progress in mathematics could readily be measured.

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Quality Control by Statistical Methods

By ELMER E. HASKINS
Air Institute of Technology, Wright Field, Ohio

INDUSTRY has always been concerned with the problem of maintaining its manufactured product at a uniform and acceptable quality level. Until the last few years, efforts in this direction usually centered around some arbitrary inspection scheme, which was without a firm foundation in theory. In recent years, however, a technique known as "Quality Control by Statistical Methods" has been widely adopted and has been found to be of tremendous benefit in maintaining desirable quality standards. This technique has a sound basis in the Mathematical Theory of Statistics. The amazing thing is that its application has been so simplified that intelligent persons with no more than a year or two of high school mathematics can use it successfully and with understanding.

There are many records of particular applications of this technique which have resulted in almost unbelievable improvements in the quality of the manufactured product. Here is an example: In August, 1945, a representative of one of the nation's leading fountain pen manufacturers, reporting at a Quality Control Society Meeting, stated that Statistical Methods, put into operation at his company, quickly reduced the per cent of defective pieces in the production of a certain part, from 7.5% defective to 2.5% defective, and within a few months, this was further reduced to only six-tenths of one per cent.

Further proof of the value of this technique to industry exists in the form of local Quality Control Societies, all of which conduct regular meetings devoted to the advancement of statistical quality control on a professional level. These groups are united under the American Society for Quality Control, which is comparable to the many national business, professional and engineering Societies, and which carries on its own publication, "Industrial Quality Control."

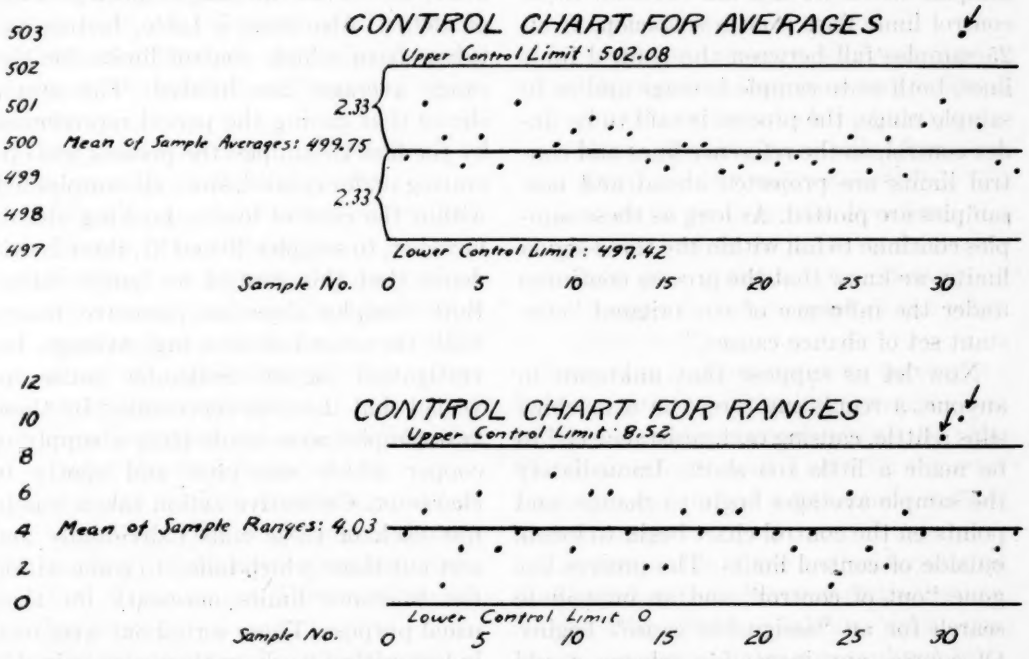
A review of a few of the concepts of elementary Statistics will be helpful in understanding this new technique. Given any frequency distribution, an efficient measure of the dispersion of the population (its tendency to spread out) is the standard deviation, or sigma. In the case of a normal distribution, for instance, a central band of two sigma width (measured one sigma on either side of the mean) will be found to include 68% of the population, a similar band of four sigma width will include 95% of the total, and a band of six sigma width will include 99.7% of all the variates. In statistical quality control, this high probability (.997) that a variate chosen at random will fall within an interval of three sigma measured on either side of the population mean, is the basis for establishing what are generally known as the "three sigma control limits."

Some understanding of this new technique can probably be obtained by considering a hypothetical application to the electrical industry. Let us assume a process which manufactures resistance coils, using automatic machinery and commercially pure copper. The process is designed to produce a coil having a resistance of exactly 500 ohms, but careful measurement of a few samples quickly reveals that the actual resistance measures all the way from a few ohms less, to a few ohms more, than the desired 500. A frequency tabulation of a sizeable number of samples (100 or more) shows the actual distribution with respect to resistance to be very close to a normal distribution. It has in fact been found to be a working hypothesis that if the manufacturing process is affected only by a constant set of chance causes, the product produced can be assumed to have a normal distribution.

The first step in setting up a "Quality Control Program" is the selection of a

sample size, which may be any number from two to twelve. For this discussion, we will assume five as a sample size. Henceforth, the word "sample" will be used to denote the aggregate, and not an individual sample. A scheme is decided upon for the random selection of samples of five according to some plan, such as one sample from every 50 or 100 produced, or one sample from each half-hour's production, and the samples are designated by number in the order in which they are drawn. Each coil is measured for its resistance, and beside each sample number is recorded the average of the five separate resistances, and the range (largest value minus the smallest) of the sample. If the five resistances of a sample are 498, 500, 501, 502 and 504, the sample average is 501 ohms, and the sample range is 6 ohms When twenty-five or more of these samples. have been collected, calculate the mean of the sample averages and the average of the sample ranges. Next, on ordinary cross section paper, prepare two graphs, one based on the sample averages and the other on the sample ranges. Using a horizontal scale which shows the chronological order in which the samples were drawn, and a vertical scale representing resistance, show on one graph the distribution of sample averages around the mean of the averages, and on the other show the distribution of the sample ranges about the average range.

Now follows a very important step, setting up the control limit lines. Here is where the mathematical Theory of Statistics is involved. First, Mathematical Statistics shows that the mean of the sample averages is the best estimate of the unknown average of the entire population from which the samples were drawn. Second, Mathematical Statistics develops a theory by means of which "sigma" can be estimated from the average range of the samples. The theory is complicated, but the application has been simplified to the point where the steps involved are merely looking in a table for a factor determined by the sample size, and multiplying this by the average range, to get sigma. In Quality Control practice, a factor is given which yields three times sigma, rather than sigma, and lines located on either side of the sample average and range reference lines, a distance of three sigma away, are the respective "control limits" already



referred to.

A consideration of the significance of the three sigma control limits will show why they are so important. If the mean of a normal distribution is known, and the sigma for the distribution of the means of samples of five is calculated, then it can be said that the probability that any sample average will vary from the distribution mean by an amount greater than three sigma, is only 0.003. As a consequence, if three sigma limit lines are added to our two control chart graphs, any of our 25 samples which do not fall between these control limit lines excites suspicion. If all 25 samples fall between the control limit lines, both as to sample average and as to sample range, the process is said to be under control, so the reference lines and control limits are projected ahead and new samples are plotted. As long as these samples continue to fall within the three-sigma limits, we know that the process continues under the influence of our original "constant set of chance causes."

Now let us suppose that unknown to anyone, a regulating screw on a machine slips a little, causing each resistance coil to be made a little too short. Immediately the sample averages begin to change, and points on the control chart begin to locate outside of control limits. The process has gone "out of control" and an immediate search for an "assignable cause" begins. Of course, any inspection scheme would have revealed a change in quality of the manufactured article, beginning at the time the screw slipped. The value of "Quality Control by Statistical Methods" is that it supplies an immediate answer based on sound theory, to the question: "Is the change in quality significant?" There are always variations in quality, but before the development of this theory, one never knew except by guess-work, when the variation was great enough to cause alarm. Now industry pretty generally relies on the three-sigma criterion, making the assumption that if an event occurs when the probability of that event is only 0.003 unless there are present unknown and undesirable causes, it is safe to assume that these undesirable causes must be present and look for them.

Charts such as would have been prepared with the example just discussed, are shown here. Suppose the original 25 samples had a mean average of 499.75 and a range average 4.03. From a table the factor 0.577, when multiplied by the average range gives 2.33. This serves to locate the control limits for averages, 2.33 units above and below the sample mean average of 499.75. Also from a table, factors are taken from which control limits for the range averages are located. The graph shows that during the period represented by the first 25 samples the process was operating under control, since all samples fell within the control limits. Looking ahead, however, to samples 30 and 31, there is evidence that this control no longer exists. Both samples show an excessive range, while the second shows a high average. Investigation for an assignable cause revealed that the coils represented by these two samples were made from a supply of copper which was poor and spotty in character. Corrective action taken was to test each of these coils individually and sort out those which failed to come within the tolerance limits necessary for their usual purpose. Those sorted out were used in less critical work, or they were paired in two's to come within tolerance limits on a job requiring a resistance of 1000 ohms.

This discussion has dealt only with the phase of Quality Control which is based on inspection by variables, that is, to determine the amount or extent of variation. Another and equally important (perhaps more important) phase deals with inspection by attributes, that is, the article inspected either passes or fails the inspection. Any discussion of this, however, would necessarily involve a discussion of acceptance sampling plans, which this article will not undertake.

The Relationship of Certain Reading Abilities to Success in Mathematics*

By EDWIN EAGLE
San Diego State College, San Diego 5, Calif.

THE RELATIONSHIP of reading abilities to achievement in mathematics is widely recognized as a topic on which additional basic research should be done. To secure some evidence in this area the present study utilized the scores, on certain standardized tests and on several specially designed tests, of 157 ninth grade general mathematics students and of 162 ninth grade algebra students of three San Diego schools.

The abilities studied in relation to achievement in mathematics were:

1. Mental age, as determined by Otis Test of Mental Ability.
2. Reading comprehension, determined by averaging Gates Reading Survey reading comprehension scores with Stanford Achievement Test paragraph meaning scores.
3. Reading speed, as measured by the Gates Reading Survey.
4. General reading vocabulary, determined by Gates Reading Survey reading vocabulary scores and Stanford Achievement Test word meaning scores.
5. Mathematics vocabulary, determined by a multiple choice test designed for this study.
6. Ability to interpret graphs, determined by tests designed for this study, one test using a line graph and a second test using a circle graph. The test included exercises in interpreting legend, axes labels, in estimating approximate values between numbered rulings, and in estimating the relative size of sectors.
7. Ability to interpret algebraic formulas, as determined by a test designed for this study which included questions pertaining to the formulas: $d=rt$, $s=16t^2$ and $S=44bd^2/l$. To correctly answer the questions the pupils had to be able to determine how one variable was affected by changes in one or more of the other variables.

Success in mathematics was determined by averaging the arithmetic reasoning and arithmetic computation scores on the Stanford Achievement Test with semester grades in eighth and ninth grade mathematics courses.

The first phase of the analysis consisted of calculating the linear correlations given in the table. Since reliability data were lacking on some of the standardized tests used the correlations given have not been corrected for attenuation. The probable errors of the correlations were approximately .05.

The mental age standard deviation was

* Based on a doctoral dissertation by the author.—Editor.

Correlation	General Math. Group		Algebra Group	
	Suc. in Math.	M.A.	Suc. in Math.	M.A.
1. Mental Age	.35		.50	
2. Reading Comprehension	.35	.76	.42	.61
3. Reading Speed	.27	.51	.18	.40
4. General Vocabulary	.25	.82	.31	.59
5. Mathematics Vocabulary	.53	.61	.48	.46
6. Interpreting Graphs	.48	.56	.38	.47
7. Interpreting Formulas	Not determined		.51	.25

1.48 for the general mathematics group and 1.11 for the algebra group, which accounts for the lower correlations for the algebra group of factors 2 to 6 with mental age, and which also makes much more significant the higher correlations for algebra students of mental age with success in mathematics.

The second phase of this study was ascertaining the effects of the specific factors, 2 to 7, on success in mathematics after partialling out mental age. To accomplish this, the general mathematics students and the algebra students were each classified into two approximately equal groups, a high and a low, on the basis of mental age. Each of these groups was in turn divided into a high and a low on the basis of each of the other factors being considered. To illustrate for the case of reading comprehension, the following groupings were made:

High Mental Age Low Rdg. Comp'n	High Mental Age High Rdg. Comp'n
Low Mental Age Low Rdg. Comp'n	Low Mental Age High Rdg. Comp'n

Due to the positive correlation of reading comprehension with mental age, this sorting on the basis of reading comprehension resulted to some extent in a second sorting on the basis of mental age. However, by eliminating a few of the highest mental age students from the "high-high" group and a few of the lowest mental age students from the "high-low" group, two groups of equal mental age means but differing in reading comprehension were secured. The means on success in mathematics, the difference of the means, and the statistical significance of this difference were then compluted. This same procedure was used for the low mental age group. This made possible the drawing of some conclusions as to the effect of reading comprehension on success in mathematics after mental age differences had been partialled out.

The effects of the other factors were analyzed in the same manner. In the case of the three factors, mental age, reading comprehension, and reading speed, a three way sorting was used to get some additional evidence on the question of the relative success in mathematics of slow readers as compared to fast readers, after partialling out both mental age and reading comprehension.

The conclusions drawn on the basis of the statistical measures derived in the above manner, together with some implications regarding the teaching of mathematics are stated in the following paragraphs.

Reading Comprehension. The positive correlation of general reading comprehension with success in mathematics appeared to be very largely associated with mental age. After sorting to partial out mental age, in each of the four pairings resulting there remain in every instance small, not statistically significant, differences in success in mathematics in favor of the higher reading comprehension groups. Expectation of improved proficiency in mathematics as a result of training to develop general reading comprehension does not seem warranted. Attention to reading problems particularly significant in mathematics, and to the mathematical aspects of general reading would seem to be the proper province of the mathematics teacher.

Reading Speed. A study of reading speed in relation to success in mathematics after partialling out both mental age and reading comprehension suggests the presence of three groups of students differing in regard to the effect of these factors. In the low mental age—low reading comprehension group the slower readers tended to be the poorer mathematics students. The inabilities and handicaps that made these students slow readers evidently also interfered with their progress in mathematics. Among the students more nearly

average in mental age and reading comprehension the slower readers tended to excel in mathematics. With these students slowness in reading appears to be associated with carefulness and precision which contribute to success in mathematics. In the high mental age-high reading comprehension group, students with high reading speed scores tended to excel in mathematics. Possibly these superior students have learned to adapt their rate of reading to the nature of the reading task, and they may therefore read more slowly when studying mathematics than they do while taking a reading test. Perhaps one of the most helpful things teachers can do in this connection is to help students learn to adapt their rate of reading to the type of material being read and to the purpose they have in mind in reading it.

General Vocabulary. Though general vocabulary correlated more highly with mental age than any other factor, it appeared to have less relationship to success in mathematics than any other factor except reading speed. There is certainly no indication that special attention to general vocabulary will have any appreciable effect on performance in mathematics.

Mathematics Vocabulary. Though the correlation of mathematics vocabulary with mental age was definitely lower than the correlation of general vocabulary with mental age, mathematics vocabulary appeared to be decidedly more important in relation to success in mathematics. After partialling out mental age it was found that those with better mathematics vocabulary scores consistently excelled in mathematics. In the light of these results it is recommended that mathematics teachers give much greater emphasis to teaching thorough understanding of the technical terms involved, for the vocabulary constitutes not only the means of communication but it is also very largely the medium of thinking and of problem solving in this area. More extensive use of concrete teaching materials seems advisable. Low test scores on questions pertaining to area and volume suggests that concrete representations of area and volume units might be used to advantage even at the high school level to teach more meaningful concepts. The names of the digits, and therefore probably the basis of our number system, were not well understood. An abacus might well be part of the equipment of the mathematics classroom, not only because of historical interest but to make more clear the basis on which our number system is built. Many students did not know the meaning of terms of geometry such as rectangle, hexagon, altitude, radius, and perimeter. The use of linkage models to present these as objects rather than as pictures, having the students handle and measure them making frequent use of the correct names might make for more effective teaching of the technical vocabulary.

The above comments serve to emphasize one important aspect of teaching mathematics, namely that in order to develop meaningful concepts pupils must have numerous concrete experiences. The suggested use of multisensory aids is for the purpose of making these experiences more real, more concrete, and more impressive. This is important, but it is only the first phase of the learning process. Under the guidance of the teacher the significant aspects of these experiences must be singled out, summarized, symbolized, and integrated into meaningful concepts and precise definitions. Pupils should be encouraged to formulate their own definitions of mathematical terms and to improve on these definitions by progressively making them more exact and more inclusive as the nature of their problems demands. They will thereby keep their mathematics on a basis of understanding and will develop greater mathematical rigor, which is largely a matter of giving formal and exact definition to concepts previously discovered intuitively. To summarize, the developing of rich mathematical concepts requires numerous concrete experiences, but this alone will not suffice. Significant

aspects of these experiences and the mathematical relationships must be discovered, brought into sharp focus and seen in relation to one another and to previously learned concepts, and understood sufficiently to serve as a sure foundation for next steps. An awareness of the importance of both of these phases of the learning process is essential.

Interpreting Graphs. Though ability to interpret graphs was less closely related to mental age than general vocabulary or reading comprehension, it was more closely related to success in mathematics than were these factors. The fact that the Otis Test of Mental Ability is largely a reading test perhaps accounts for this. After partialling out mental age this positive relationship persisted, especially with the general mathematics group. Graphical representations picture mathematical relationships more directly than do ordinary sentences, and they may therefore be peculiarly well suited to presenting mathematical explanations and for enabling students to think through mathematical problems. They may in many cases serve as convenient stepping stones to the use of the more abstract symbolism of the algebraic formula or equation.

Interpreting Formulas. Though ability to interpret formulas did not correlate highly with mental age, with the algebra students it correlated higher than any other factor with success in mathematics. (The general mathematics students did not take this test.) The lowness of the correlation with mental age is not surprising when we stop to consider that the meanings of the mathematical conventions are largely the results of arbitrary choice rather than of logical reasoning. An awareness of this on the part of mathematics teachers is important so that they will consciously and thoroughly teach the meanings of the mathematical conventions rather than assume the pupils know them or suppose that understanding of these conventions will develop automatically.

Relationship of Methods of Presenting Data to the Process of Problem Solving. The results of this study regarding the importance of reading abilities as they relate to the interpretation of quantitative data presented by various methods, suggest that possibly the content of mathematics courses should be selected and organized to give more emphasis to mathematics as a method of organizing and presenting data, to the various methods of doing this, and to the relationship of these methods to one another and to the process of problem solving. These methods may be classified as: the sentence; the mathematical table; the graph, including the bar graph, line graph, circle graph, the diagram, the map, and the three dimensional model; and the formula or equation.

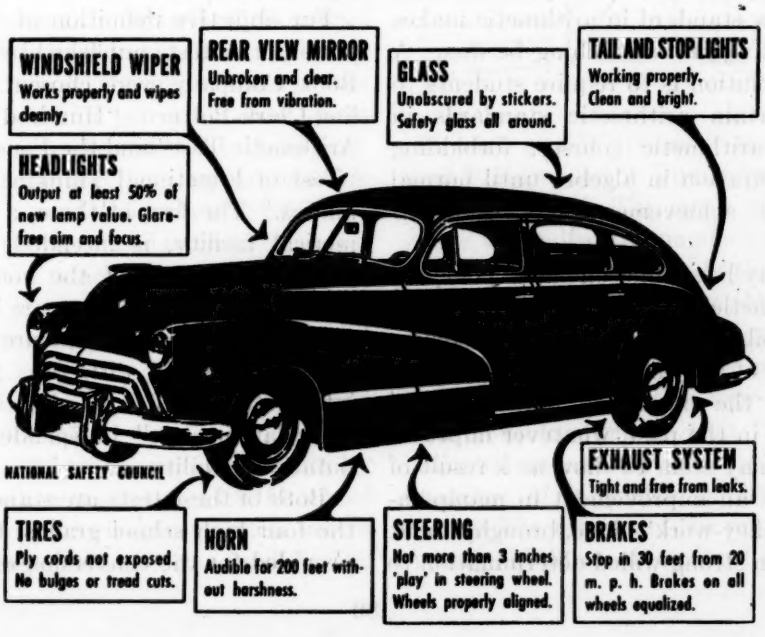
Highly abstract symbols, namely words and number symbols, are used in the sentence to communicate quantitative ideas. The order and arrangement of the words and numbers is determined by rules of sentence structure rather than by any quantitative or logical relationship. Therefore the sentence, though the most familiar basis of presenting data, is not most effective from the mathematical standpoint. In the mathematical table the same symbols, words and number symbols, are used; but the punctuation marks are replaced and improved on by rulings making rows and columns, and the order and arrangement derives from the logical and quantitative relationships involved rather than from rules of sentence structure.

The various types of graphs make use of a more direct type of symbolism than we find in the sentence or in the mathematical table. Graphs picture the essential relationships more directly, more vividly, and more logically. Something of the relative size, directional aspects, and positional relationships is part of the visual impression. In other words, the graph improves on the sentence by a more direct and more effective symbolism, and also by a more systematic and more logical arrangement.

In the formula or equation we have the most concise, highly concentrated symbolism, with all non-essential data eliminated, and with the mathematical relationships clearly indicated to one familiar with the mathematical conventions. Also, the data have been put in a form to which the mathematical processes may be directly applied, thereby making shorter and easier the process of solving the problem. This enables us to readily solve many problems which would otherwise be difficult or impossible, for it permits us to substitute a systematic generalized procedure which can be learned for the more difficult process of reflective thinking.

In mathematics classes problems are usually presented to students in the form of sentences. Efforts should be made to have the problems arise out of situations more real and more vital to the pupils, but even then the data would in most cases appear first in sentence form. Too often we have the student attempt to go directly from the sentence to the equation or formula. The other methods of organizing quantitative data would often serve as helpful intermediate steps in the process. Frequently the first step might well be to arrange the essential data in table form; this for one thing would assure careful reading of the problem, and it would make more apparent the relationships of the various quantities involved. Then if the equation or formula is not evident a diagram or graph may serve to indicate more clearly the relationships on which the equation must be based. Thus all of the four methods of organizing data, the sentence, the table, the graph, and the formula, may serve as steps and as definite aids in the process of problem solving.

This function of selecting and organizing the essential data is important in life situations to a greater extent than in most textbook problems. In the problems of the textbook much of this necessary phase has already been completed. More emphasis on this aspect of mathematics would relate the work of the schools more closely to practical applications, thereby making the mathematics courses more functional and probably better motivated. In such courses reading abilities, such as those considered in this study, would be of even more importance in relation to achievement in mathematics, and they would therefore be developed to a greater extent than is the case in the usual mathematics course.


WINDSHIELD WIPER
Works properly and wipes cleanly.
REAR VIEW MIRROR
Unbroken and clear. Free from vibration.
GLASS
Unobscured by stickers. Safety glass all around.
TAIL AND STOP LIGHTS
Working properly. Clean and bright.
HEADLIGHTS
Output at least 50% of new lamp value. Glare-free aim and focus.
NATIONAL SAFETY COUNCIL
EXHAUST SYSTEM
Tight and free from leaks.
TIRES
Ply cords not exposed. No bulges or tread cuts.
HORN
Audible for 200 feet without harshness.
STEERING
Not more than 3 inches 'play' in steering wheel. Wheels properly aligned.
BRAKES
Stop in 30 feet from 20 m. p. h. Brakes on all wheels equalized.

A Study in the Growth of Arithmetic Power

By WILLIAM LEE
Santa Monica High School, Santa Monica, California

THE EXTENT to which high school courses such as algebra and geometry contribute to the pupil's arithmetic power is frequently questioned by those who are not themselves teachers of mathematics. Even experienced mathematics teachers often are prone to list retention of arithmetic, rather than increased power in arithmetic, as one of the desirable outcomes of such courses.

No doubt this is due to the habit of "compartmentalizing" mathematics. Organization of subject matter is of course necessary and desirable; but it should be made to emphasize rather than to eclipse the fundamental relationships of all mathematics. The understandings developed in algebra have their special applications in arithmetic; for the average pupil, perhaps this is their most important application. The entities studied in geometry are susceptible to arithmetic treatment, and relationships developed geometrically can throw new light on the way arithmetic numbers behave.

The fact that the pupils pouring into our high schools year after year are woefully below standard in arithmetic makes it imperative that something be done. A popular solution is to require students to attain certain arithmetic standards in remedial arithmetic courses, forbidding their registration in algebra until normal arithmetic achievement has been attained.

This may be an uneconomical policy. If the arithmetic course which is offered to bring pupils up to standard is merely a repetition of subject matter and methods to which the pupil has not responded favorably in the past, whatever improvement he may seem to show as a result of it may be an improvement in manipulative "monkey-work" alone, brought about by his own strong-willed determination to improve, but unaccompanied by the increased understanding which alone could make his manipulative skill of value to him. Possibly he would derive more practical and lasting benefit by taking a new view of arithmetic processes in the light shed on them, for instance, by an understanding of the equation method and the fundamental processes of algebra.

This is not to advocate that all pupils be admitted indiscriminately to the intensive, highly technical algebra course; but rather that, for those deficient in arithmetic achievement, a course be made available which shall be conceived less as "remedial" than as "re-view," in the sense of a *new view*, a new perspective, which may prove more fruitful than the often somewhat limited view already presented during the pupil's previous school experience.

To find out to what extent the various high school mathematics courses develop power in arithmetic, the author undertook a testing program in a small (average daily attendance about 500) rural California high school.

For objective definition of "arithmetic power," two tests published by The World Book Company were chosen: the Schorling-Clark-Potter "Hundred Problem Arithmetic Test" and the Foust-Schorling "Test of Functional Thinking in Mathematics." The first of these, a test of numerical facility, is intended to "give a practical measure of the computational skills which experts agree are basic." The other is designed to "measure a student's power in dealing with (i.e., recognizing, interpreting, and expressing) mathematical relationships," independent of computational ability.

Both of these tests are standardized for the four high school grades, tables being provided for the conversion of raw scores

into percentile ranks for each grade. However, since the standardization scores were obtained at mid-year, while the tests were administered to the experimental classes in the fall and in the spring in order to measure change, it was necessary to translate the published norms into fall and spring standards for each grade. Thus, if a raw score of 23.5 represented 50th percentile at midyear 9th grade, and 25.5 represented 50th percentile for midyear 10th, and 28.5 represented 50th percentile for midyear 11th grade, it seemed fairly reasonable to suppose that the typical median 10th-grader in the standardization group progressed from (23.5+25.5)/2 or 24.5 raw score to (25.5+28.5)/2 or 27 raw score during the course of the school year. By the published table of norms, raw score 24.5 corresponds to 45th percentile for 10th grade, while 27 corresponds to a 10th grade percentile rank of 55. Thus the median 10th grade pupil (50th percentile at midyear) would be expected to rank at the 45th percentile at the beginning of the year, and at the 55th percentile at the end: an increase of 10 percentile points during the course of his school year.

According to the published tables of norms, uniform gains were not made at all ability levels in the two tests by pupils in the various grades. In fact there are instances where the hypothetical normal pupil did less well in the 10th grade, for instance, than in the 9th. However, the average change in percentile rank, from the beginning to the end of the school year, in the two tests together, appears to be an increase of some 4 or 5 percentile points.

Each of the tests used is available in two forms, one of which was used in the fall and one in the spring. Practice effect was further reduced by the fact that most of the pupils, except the freshmen, had previously taken one form of the test during the year 1944–45.

Figure 1 shows the progress made by various mathematics classes for the school year 1945–46. In computing these scores the following procedure was used: in the

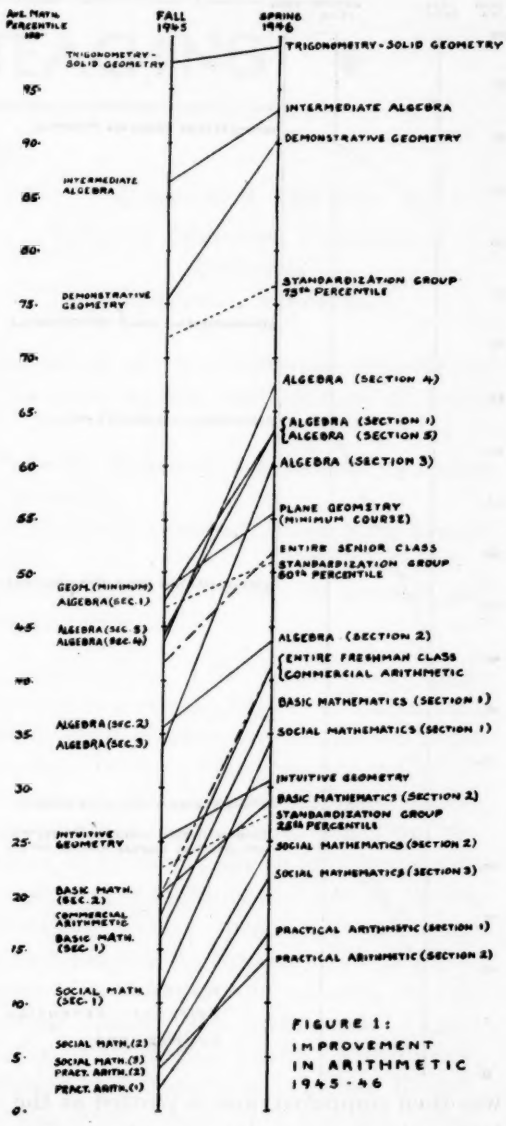


FIGURE 1: IMPROVEMENT IN ARITHMETIC 1945-46

fall, each pupil's percentile rank in Hundred Problem and in Functional Thinking was determined. The mean of these two scores was called the pupil's Fall Average Mathematics Percentile. At the end of the year, his Spring Average Mathematics Percentile was computed in a similar way. Pupils for whom both a fall *and* a spring score were not available were eliminated from consideration, so that the changes indicated are entirely uninfluenced by the dropping of incapable pupils from the classes. The median of the remaining Fall Average Mathematics scores for each class

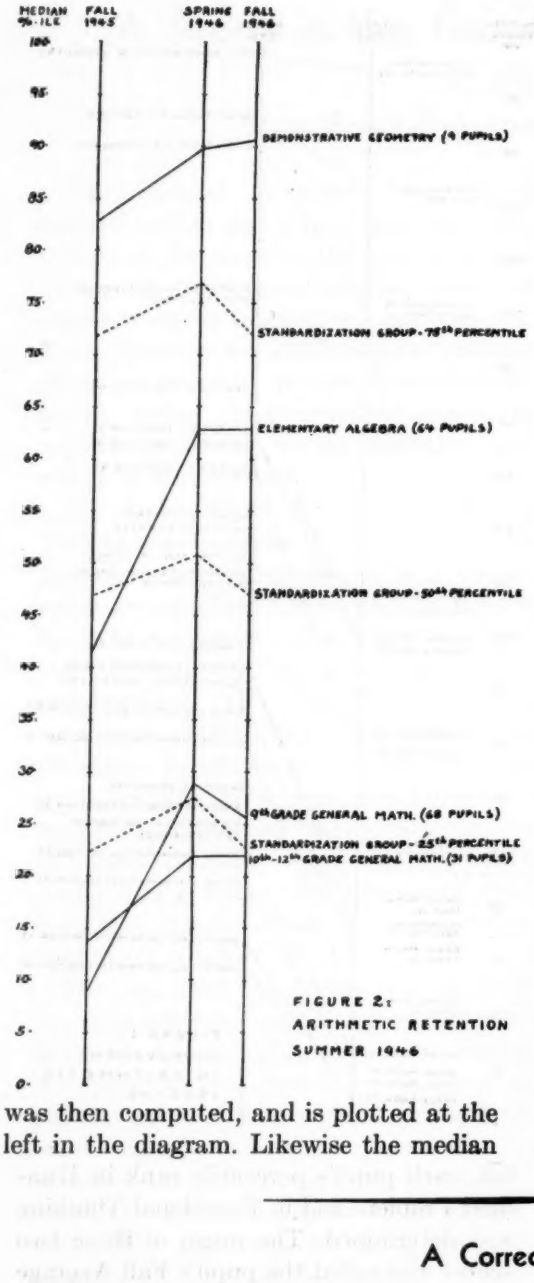


FIGURE 2:
ARITHMETIC RETENTION
SUMMER 1946

was then computed, and is plotted at the left in the diagram. Likewise the median for each class in the spring is plotted at the right. The line connecting fall and spring medians for a given class in the diagram indicates by its slope the growth in arithmetic power evidenced by that class in the course of the year.

It will be noted that *every class* showed more gain than did the standardization group (which included all pupils in each grade, whether studying mathematics or not), with the exception of the class in Solid Geometry and Trigonometry, which was already so near the top of the scale that it had little room for improvement.

Lest it be supposed that these gains may have been lost by forgetting during the ensuing summer vacation, the median scores of those pupils who took the tests in Fall '46 as well as in Fall '45 and Spring '46 are plotted in Figure 2.

A justifiable conclusion would seem to be that the various high school mathematics courses can be taught in such a way as to contribute effectively to the development of functional ability in arithmetic; an ability which, grounded in understanding, will be retained. A reasonable hope is that the individual will find this ability useful in practical situations which may seem far removed from the specific subject matter of, say, algebra or geometry. Lastly, it is fondly believed that the progress of mathematics pupils in other respects vital to the welfare of society, if objectively measurable, might prove to be as great.

A Correction

On page 85 of the February issue of THE MATHEMATICS TEACHER the address of *Knowledge Builders* was incorrectly given. It should have been 625 Madison Avenue, New York 22, N. Y.—EDITOR.

◆ AIDS TO TEACHING ◆

By

HENRY W. SYER
School of Education, Boston University
Boston, Massachusetts

DONOVAN A. JOHNSON
College of Education, University of Minnesota
Minneapolis, Minnesota

BOOKLETS

B. 5—*Mathematics and the Laws of Nature* by Hermann Weyl
United States Rubber Company, Rockefeller Center, New York, New York
Free booklet; 4 pages.

Description: This is a reprint of the talk given by Dr. Weyl, of the Institute for Advanced Study, on the New York Philharmonic Symphony radio program on February 23, 1947. He covers the development and importance of geometry, numbers and the application of mathematics to physical laws. Examples are drawn from Anaxagorus, Galileo, Kepler, Pythagorus, Newton and Schrodinger.

Appraisal: Every short, simple summary of mathematics serves to strengthen the belief that there is no private and royal road to geometry or any other important branch of mathematics. Dr. Weyl's summary is brilliant, clear and illuminating in proportion to the amount of mathematics that the reader brings to it. A roomful of non-mathematicians, after hearing the original broadcast, said that it was interesting and that many of the words sounded familiar, but that they did not understand much of it, nor could they reproduce the examples and arguments that he used. The printed copy, which can be mulled over and studied, will serve much better.

If enough copies can be supplied for the whole class to read outside and then class time be given to discussion, with the instructor illustrating points mentioned in the speech, this booklet would be a tremendous force to motivate biographical, historical and mathematical projects by the more mature mathematics students.

B.6—*He Measured in Millionths* by James Sweinhart.
Ford Motor Company, Dearborn, Michigan.
Free booklet on history of science and measurement.

Description: This twenty-four page booklet is a sketchy history of science and particularly of measurement. Eleven pages contain this story before getting to Carl E. Johansson who invented the gage blocks carrying his name. Eight pages are devoted to his story, one to his picture, two to pictures of factories and two to covers. Practically every page contains some sketch illustrating the story. To give you some idea of the ground covered, the following names are mentioned: Henry Fabre, Pythagorus, Euclid, Archimedes, Hipparchus, Copernicus, Galileo, Kepler, Pascal, Snell, James Watt, Galvani, Volta, Ampere, LaPlace and Edison!

Appraisal: This little booklet tries to do too much: the treatment of science and measurement is sketchy, disjointed and unorganized. There are too many assertions of the contributions which these men made without supporting evidence in the form of examples of their discoveries or their methods. There is much too much emphasis on the work of Johansson, even if the book is his story. In that case the long introduction of the history of science should have been omitted. However, if used critically, this book can be very help-

ful as supplementary material in physics and mathematics.

EQUIPMENT

E.3—*Blackboard Stencils*
Corbett Blackboard Stencils, 548 Third Avenue, North Pelham 65, New York.
Stencils for producing temporary graph outlines for rectilinear and polar graphs. $12.45 each.

Description: These flexible charts are made of varnished cloth, mounted on a spring roller and enclosed in an oak frame. By pulling down the chart as one would a window shade and then rubbing a used blackboard eraser over the surface one can stencil small dots onto the blackboard through the holes in the chart. When the chart is rewound into its frame, the dots can be seen on the blackboard forming the outline of the lines for a graph. Either rectilinear or polar coordinate graphs can be supplied. The graphs are about a yard square.

The rectilinear graph is 24 squares by 24 squares, each $1\frac{1}{2}''$ wide and with 5 holes to the square. This allows subdivision of the square into 10 parts to be estimated. The polar coordinate graph is 35″ in diameter and has central angles of 15° and 5°. There are ten circles $1\frac{3}{4}''$ apart and the outer circle has 360 dots around its circumference.

Appraisal: These charts are a big improvement over the older method of scratching lines into the blackboard to make a rectilinear graph. That method was so permanent that it made the board undesirable for other uses; it was confusing to write on; therefore it was usually on a side board, inconvenient. This new stencil makes it possible to have graphs at the front of the room and yet not tie up any board space permanently. However, the stenciled graph is not so clear and distinct as the one cut into the surface, and it is erased if part of the figure is erased so that it is more difficult to shift the position of a graph with respect to the axes.

FILMS

F. 7—*Congruent Figures*
Knowledge Builders, 625 Madison Avenue, New York 22, New York; Producer: John R. McCrory Studio; Collaborator: R. C. Jurgenson
16 mm. sound film; 1 reel; black and white; 1945.

Content: By showing the need for the manufacture of identical parts in mass production factories, this film emphasizes the reason for studying congruency. Congruence is then defined by showing the comparison of triangles of different shapes and sizes, in each case congruence is tested by superposition. The three common theorems on congruence of triangles (SAS = SAS; ASA = ASA; SSS = SSS) are "proved" by superposition. A typical exercise using congruence is then proved formally by listing the steps and the reason for each.

Appraisal: Although this film shows congruency applied in industry and shows clearly the process of superposition, most of the film discusses blackboard drawings and proofs in a traditional manner. It is the opinion of the reviewer that this film does nothing that could not be done as well or better by the typical mathematics teacher using the blackboard for illustration. And if the teacher has triangles for illustration, drawing and manipulation by students, learning will probably be more effective than with the use of the film. The film could well have placed most emphasis on showing the applications of congruency rather than doing what every geometry textbook does using similar drawings and notation. The commentary is clear, but slow, and at times lags behind the action on the screen. The photography, sound, and animated drawings are satisfactory.

F. 8—*Locus*
Knowledge Builders, 625 Madison Avenue, New York 22, New York.
16 mm. sound film; 1 reel; black and white; 1944.

Content: The path of a baseball, the course of a ship, and the trajectory of a tracer bullet are used to give meaning to the term locus. By animated drawings, this film illustrates the following loci:

1. the locus of points two feet from a point is a circle
2. the locus of points one foot from a given line gives two parallel lines
3. the locus of points equidistant from two points is the perpendicular bisector of the the line joining the two points
4. the locus of points equidistant from the sides of an angle is the bisector of the angle
5. the locus of the center of a circle rolling around a rectangle
6. the locus of points within a rectangle is the area of a rectangle
7. the locus of points two feet from a point and one foot from a given line is the point of intersection of a circle and a line
8. the locus of points equidistant from two parallel lines is a line parallel to the two lines and midway between them
9. the locus of points one foot from a circle whose radius is one foot is a circle and a point

Locus is defined as including all points and only those points that satisfy the conditions described. The film ends with a brief summary.

Appraisal: The animated drawings used in this film should contribute to development of the understanding of locus which is often difficult to clarify in the classroom. The drawings are clear and well labeled. The film could be improved by the inclusion of many more practical applications of loci. The commentary is quite slow and does not always coincide with the pointer in the pictures.

F. 9—*Lines and Angles.*
Knowledge Builders, 625 Madison Avenue, New York 22, New York.
16 mm. sound film; 1 reel; black and white; 1944.

Content: Geometry is presented as an ancient science that is used to survey mountains, roads and dams; used to plan cities, pyramids, and skyscrapers; used to level masonry work and Grecian structures.

A carpenter's square and T-square are used to draw plumb lines and parallel level lines; the plumb line is perpendicular to the level lines. These parallel lines merge to illustrate that only one straight line can be drawn between two points, and parallel plumb lines merge to show that at a given point on a given line there can be only one perpendicular drawn.

A protractor is made by dividing a circle into four arcs by a draftsman's triangle and then each of the arcs divided into 90 arcs, each arc representing one degree. This protractor is then used to represent 15°, 30°, 45°, and 90° angles. Other angles are drawn to illustrate acute, complementary, supplementary, obtuse, conjugate, and reflex angles. The angles illustrated are labeled with a dot between the sides rather than by the conventional arc.

F. 10—*Quadrilaterals*
Knowledge Builders, 625 Madison Avenue, New York 22, New York.
16 mm. sound film; 1 reel; black and white; 1944.

Content: By animated drawings this film shows a great variety of quadrilaterals and the relationships that exist between sides, angles, and diagonals. The drawings show how quadrilaterals change from one type to another. Terms such as trapezium, quadrangle, rectangle, trapezoid, rhombus, and square are defined and illustrated by drawings. Short oral proofs are given of relationships, such as: diagonals of a parallelogram bisect each other; opposite angles of parallelograms are equal and adjacent angles are supplementary; diagonals of a rectangle are equal; diagonals of a rhombus are perpendicular to each other and bisect the angles. A few applications in real life are pictured by showing the rectangles in architects' building plans and the rectangles in skyscrapers. Some of the terms used such as trapezium are not commonly accepted today.

F. 11—*Similar Triangles*
Knowledge Builders, 625 Madison Avenue, New York 22, New York.

16 mm. sound film; 1 reel; black and white; 1945.

Content: The concept of similarity is introduced by showing a photograph and its enlargement, pointing out the equality of angles and the proportionality of corresponding parts. Similarity is formally defined and its use illustrated by the solution of a shadow problem. The difference between similar triangles and congruent triangles is shown. By informal proofs or superposition, triangles are proved similar if the angles of one equal the angles of the other or if the sides of one are proportional to the sides of the other. Proportionality of altitude, medians, and perimeters are "proved" informally. The proportionality of similar triangles is then used to prove that the product of the segments of a chord of a circle equals the product of the segments of the intersecting chord.

Appraisal of F. 9, F. 10, and F. 11: These films will furnish the geometry teacher with a different way of presenting material that is often uninteresting to secondary pupils. A motion picture can be an effective method of showing applications of geometry, or to show equality by superposition, or to show how a figure can be changed. These films do these things, but the drawings and commentary is much too similar to a typical class room presentation of the same material on the blackboard. Most geometry teachers would like more emphasis on the applications of the principles presented rather than proofs of theorems included in the geometry textbook.

Technical Qualities: Photography: Good. Drawings are too similar to blackboard illustrations. Commentary: Clear. Slow, lags behind screen action. Content: Usual textbook material.

FILM-STRIPS

FS. 3—*Problem Analysis.*
Jam Handy, Detroit, Michigan

Description: This film-strip illustrates and discusses the steps to follow in the solving of problems that are mathematical as well as non-mathematical. It suggests the following steps in solving life problems: (1) organize the facts, (2) draw on experience, (3) formulate the approach. A lever problem is used to illustrate the value of a drawing, the expression of known facts, the use of a formula to form an equation, and the final solution of a mathematical formula.

Appraisal: Problem solving is always a difficult skill to teach and, consequently teachers will want to use every possible aid available. Except for the pictures that emphasize the importance and solution of life problems, this film-strip contains little that is not already found in mathematics textbooks. As in most of the film-strips of the Jam Handy Kit on Mathematics, this one contains too much reading material.

Technical quality: Photography and drawings: fair. Content: Too much written material. Level: Ninth grade algebra.

INSTRUMENTS

I. 4—*Observoscope* (Star Finder and Directional Instrument).
Millar Instrument Company, Inc.; Post Office Box 64; Newark, N. J.
Instrument for field exercises. $12.50, plus shipping charges.

Description: This simple, yet accurate, instrument is made of black plastic and stands seven inches high and four and a half inches wide. The hollow tube which acts as a sighting device is so pivoted that it may be adjusted and set in three different dimensions: latitude, azimuth, and declination. There are essentially two ways that it can be used: for measurement or location. Angular locations on the celestial sphere or terrestrial globe can be measured; or, by means of a table of angular values of sidereal hour angles and declinations, the instrument can be set so that it is pointing toward a desired star or constellation. The chief measurements which are described are the following: altitudes, azimuths or bearings, local civil time, and direction of true north.

Appraisal: The instrument is small and yet quite sturdy. It is not designed with sufficient accuracy to be called a scientific instrument, but it serves to illustrate the principles better than one with more refinements. The proper place for such a tool is in the senior mathematics work when plane geometry, solid geometry, and trigonometry are combined into elementary celestial navigation, geography and easy astronomy. If these are not part of the present mathematics course; then this instrument will serve as one more means and motivation for throwing out some old material and teaching more applications.

1.5—*Ohmite Ohm's Law Calculator*

Ohmite Manufacturing Company; 4835 Flounoy Street; Chicago 44, Illinois.

Cardboard slide rule for making electrical calculations. $.25.

Description: Three openings on the front of the calculator contain six pairs of scales which can all be set together. In other words, if a pair of values are alined on any pair of scales, all the other pairs may also be read with corresponding values. The six pairs of scales are as follows: (1) watts-volts, (2) milliamperes-watts, (3) volts-milliamperes, (4) Ohms, against a constant mark, (5) watts-amperes, and (6) amperes-volts. On the reverse side of the rule, three small openings aid in finding the stock numbers of fixed resistors, adjustable resistors and rheostats made by the company furnishing the rule.

Appraisal: The very fact that this device is a commercial product made for practical electrical use and supplied by an industrial firm is a large point in its favor. Here, then, is mathematics being used outside the classroom; here is a slide-rule device made for practical electrical workers and all based on the principle of logarithms. What could be more convincing?

Along with the teaching of formulae (which are printed on the face of the rule) and the use of logarithms, slide rules and other aids to calculation, this calculator could be used in the algebra classes. It is simple, small and inexpensive enough so that every algebra teacher could have one as part of the permanent, classroom collection of aids.

PLANS FOR CONSTRUCTION

PC. 3—*How to Build Terrain Models*

Superintendent of Documents; U. S. Government Printing Office; Washington, D. C.

Booklet giving directions for terrain model construction; 28 pages; $.10.

Description: This booklet was originally prepared for training of naval personnel during the war, but it has so many educational uses that the Office of Education has cooperated to have it reprinted for general use. It covers the following topics: Gathering information, what to do with the information (including coordinates, nautical miles and string grid), scale, enlargement (by projection, grid enlargement or by pantograph), building the contours, covering the contours (including five formulas for plaster mixtures), and methods of obtaining texture, color, atmosphere and water effects. The back of the booklet has tables of metric linear measure and scale equivalents. The whole booklet is filled with pictures and drawings so as to make the whole process very understandable.

Appraisal: Do not appear amazed at the suggestion that such a booklet should be used in a mathematics class. If this class took on the project of constructing a terrain model of the city or town in which the school was located, of the nearby country, of some development being conducted nearby, such as flood control, water power planning, soil conservation, air transportation study, town or road planning, or farm resettlement; then the class would find there was plenty of mathematics learned. These topics are included: latitude, longitude, great circles, statute miles, nautical miles, representative fraction, graphic scale, bar scale, dividing a line into a given number of equal parts, latitude and longitude scales, degrees-minutes-seconds, multiplying and dividing

both sides of an equation by the same quantity, elevations, contour lines, plus and minus numbers (above and below sea level), vertical exaggeration, metric system, proportional lines, similar figures, theory of pantograph, proportions in mixing formulas, proportions of color in color mixtures, relationship between appearance and estimation of distance out of doors and also under water. Is that a convincing enough list to justify time in a mathematics class?

Correction in E.1, page 83, February issue: Math-O-Felt contains 19 sets of figures, over 80 figures in all; not 19 figures.—Editor.

A Toast to Teachers of Mathematics

To you—who lead the youth of our nation in clear, postulational and observational thinking to see, as Whitehead says, "what is general in what is particular, and what is permanent in what is transitory"—

To you—who stand on a platform dedicated to freedom of thought and to understanding of the laws of the finite and the infinite and dedicated to learn without fear or favor of the possible, the probable, and the actual relationships in our work-a-day world and the roles played by the actors in God's nightly drama—

To you—who excel in your methods of establishing and of exhibiting truth—

To you—who serve your fellow man with fidelity, with meekness, and with efficiency—

To you—be praise for your intelligence
honor for your integrity
satisfaction for your zealous efforts, and
security for your unselfishness.

W. S. SCHLAUCH

◆ THE ART OF TEACHING ◆

A New Technique in Plane Geometry

By D. A. ZARLENGO
East High School, Denver, Colo.

WITH the ending of the war, many of us feel that the world and the life of earlier years will never return. The rapid developments in all fields during the past several years will leave their results with us, and we shall feel and live them long into the future. Education has had its share in this transition. During the training of American youth all methods of presentation of subject matter had to be tried for the quickest and best method. Needless to say visual instruction received a major emphasis; enough to make it a definite part of modern teaching.

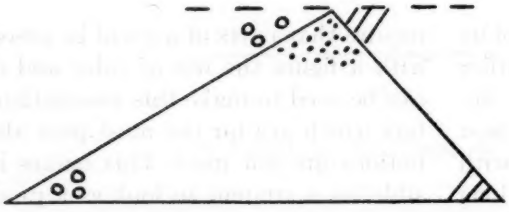
With this in mind, the visual presentation of plane geometry was begun by the author, and now a rather complete work has evolved. Of course the visual idea in geometry is not new. Most teachers, when presenting their material, make use of some form of it, especially when they wish to bring out a comparison between parts that go to make up proofs both in figures and in the proof proper. Also, many of the text books that were reviewed during the development of color proofs were found to make partial use of design and other visual notations; but it was evident that these aids were not used for complete comprehension of a proof. They were used rather, to indicate a special relationship between the parts in a figure only.

In teaching geometry an attempt was made to go beyond the partial use of visual aid in demonstrating geometric relationships. A complete proof through visual aid such as color and design was brought out. The effect was well received and the results were gratifying.

Since the demonstrations in geometry require that parts of a proof be associated with a figure the use of color and design can be used to make this association. Letters which are for the most part abstract notions are not used. This makes it possible for a student to look at a proof and see immediately in what portions of the given figure equal angles and lines fall since the angle symbol in the proof proper bear the same color as to those to which they are related in the figure. Thus, the relationship goes from the eye to the mind without the necessity of locating letters or seeking the relationship between letters and the proof.

The effectiveness of the color proofs was evident with the first few that had been worked out. To make a more accurate and conclusive study, seventy-five theorems covering the most fundamental concepts were drawn and mimeographed for pupil use. The method of study followed a pattern fast becoming popular. First in the presentation is the introduction of new ideas and principles. This would be the color proof. As an example let us take this statement: "The sum of the angles of a triangle equals a straight angle." The figure for this is the typical triangle with the construction line at the top vertex parallel to the base. One pair of alternate interior angles is colored yellow, the other, red. The vertex angle is blue and lies between a yellow and a red angle. The pupil readily sees that the red plus the blue plus the yellow give a straight angle at the top of the triangle, and thus concludes that the same is true for the three angles of the triangle since amounts may be substituted for their equals in any problem.

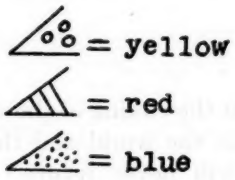
The second step involves comprehension. Here the figure is redrawn but the colors are omitted and letters are placed at the proper points. The pupil is then requested to write a complete proof in letters but following the ideas behind the colors. In doing this the understanding of the proof as obtained from the color demonstration is applied to the necessary letter comprehension.



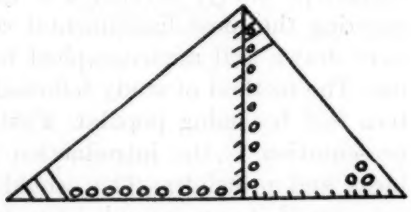
In the third step exercises involving the statement are given and in these some color relationships are made. The pupil can test his comprehension of the proof through this step.

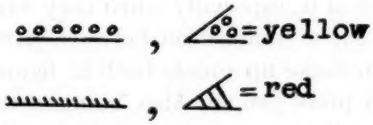
The final step brings out the practice idea. This is accomplished through problems involving not only the statement proper but also all previous usable statements.

The colored figure would appear as follows:



The effectiveness of this visual method is especially valuable when statements in proportions are discussed. Since the proportional sides lie opposite the respectively equal angles these sides will bear the same colors as the opposite angles. Thus: in proving that the altitude of a right triangle is the mean proportion between the parts of the hypotenuse, a figure as follows is used.





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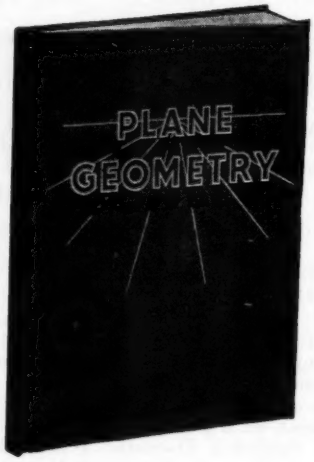

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